7.4 Flat Belts
1. Determine the minimum number of turns of rope that will allow the 5-lb force to support the 600-lb block, if the coefficient of static friction is 0.15.

(Tensions in the rope)

1. Impending motion
(The motion can't be up, since the 5-lb force is too small to lift the 600-lb block.)
3. Apply the equation for belt friction:

\[ T_2 = T_1 e^{\mu \beta} \]  

(1)

In this equation, \( T_2 \) is the tension in the direction of impending motion

\[ T_2 = 600 \text{ lb} \]

The other tension, \( T_1 \), is in the direction opposite the impending motion, so

\[ T_1 = 5 \text{ lb} \]

4. Since \( \mu = 0.15 \), Eq. 1 becomes

\[ 600 \text{ lb} = (5 \text{ lb}) e^{0.15 \beta} \]

Solving gives \( \beta = 31.917 \) radians. If \( n \) is the number of turns of rope, then the number of radians is \( 2\pi n \). Equating this to \( \beta \) gives an equation for \( n \):

\[ 2\pi n = \beta \]

\[ = 31.917 \text{ rad} \]

Solving for \( n \) gives

\[ n = 5.08 \text{ turns} \]

Rounding off to the next higher integer then gives

\[ n = 6 \]

←Ans.
2. If the coefficient of static friction between the rope and the fixed circular drums A and B is 0.2, determine the largest value of the force $P$ that can be applied without moving the 150-lb weight upwards.

Because slip impends between the rope and the drum, we can apply the equation for belt friction:

$$T_2 = T_1 e^{\mu \beta} \quad (1)$$

where $T_2$ is the tension in the direction of impending motion. In our particular problem,

$$T_2 = T_{AB}$$

and the tension opposing the impending motion is

$$T_1 = 150 \text{ lb}$$

Using these results and $\mu = 0.2$ and $\beta = \frac{\pi}{2}$ in Eq. 1 gives

$$T_{AB} = (150)e^{0.2(\pi/2)}$$

$$= 205.4 \text{ lb}$$
Tensions in the rope on either side of drum B

Impending motion

\[ T_{AB} = 205.4 \text{ lb} \]

Flat-belt friction equation:

\[ T_2 = T_1 e^{\mu \beta} \]

so,

\[ P = (205.4 \text{ lb}) e^{0.2\beta} \quad (2) \]

Geometry

\[ \beta = 60^\circ = \frac{\pi}{3} \text{ rad} \]

Using \( \beta = 60^\circ = \frac{\pi}{3} \text{ rad} \) in Eq. 2 gives

\[ P = (205.4)e^{0.2(\pi/3)} \]

\[ = 253 \text{ lb} \quad \text{Ans.} \]
7.4 Flat Belts Example 3, page 1 of 3

3. Determine the smallest force $P$ applied to the handle of the band brake that will prevent the drum from rotating when the 15 lb-ft moment is applied. The coefficient of static friction is 0.25, and the weight of lever arm ABC can be neglected.

![Diagram of drum and belt system]

1. Free-body diagram of drum

2. Impending slip of band relative to drum (An observer on the drum would see the belt move down.)

3. Equilibrium equation for drum

$$\sum M_D = 0: T_A(6\text{ in.}) - T_B(6\text{ in.}) - 180\text{ lb} \cdot \text{in.} = 0 \quad (1)$$
The belt-friction equation is in general

\[ T_2 = T_1 e^{\mu \beta} \quad (2) \]

where \( T_2 \) is the tension \textit{in the direction of impending slip}. In our particular problem,

\[ T_2 = T_A \]

and the tension opposing the impending motion is

\[ T_1 = T_B \]

Eq. (2) becomes

\[ T_A = T_B e^{0.25(3.316)} \quad (3) \]

Solving Eqs. 1 and 4 simultaneously gives

\[ T_A = 53.237 \text{ lb} \]
\[ T_B = 23.237 \text{ lb} \]

\[ \beta = 180^\circ + 10^\circ \]
\[ = 190^\circ \]
\[ = \frac{190}{180} \pi \text{ rad} \]
\[ = 3.316 \text{ rad} \]
7.4 Flat Belts Example 3, page 3 of 3

6. Free-body diagram of lever arm ABC

\[ T_A = 53.237 \text{ lb} \quad T_B \sin 80^\circ = (23.237 \text{ lb}) \sin 80^\circ \]

\[ \begin{align*}
T_A & = 53.237 \text{ lb} \\
T_B \sin 80^\circ & = (23.237 \text{ lb}) \sin 80^\circ
\end{align*} \]

7. Equilibrium equation for lever arm

\[ \sum M_A = 0: (23.237 \text{ lb}) \sin 80^\circ)(10 \text{ in.}) - P(10 \text{ in.} + 15 \text{ in.}) = 0 \quad (5) \]

Solving gives

\[ P = 9.15 \text{ lb} \quad \leftarrow \text{Ans.} \]
4. The uniform beam ABC weighs 40 lb. The coefficient of static friction between the cord and the fixed drum D is 0.3. Determine the smallest value of the weight W for which the beam will remain horizontal.

1. Free-body diagram of block B

2. If end A of the beam is about to move down, then block B is about to lose contact with the beam. Thus the normal force N is zero.

3. Equilibrium equation for block B

\[ \sum F_y = 0: \ T_B - W = 0 \]
7.4 Flat Belts Example 4, page 2 of 3

4) Free-body diagram of beam

5) \( T_A \) is the tension in the cord.

6) The 40-lb weight of the beam acts through the center of the span.

7) Equilibrium equation for the beam

\[ \sum \mathbf{M}_C = 0: \quad (40 \text{ lb})(4 \text{ ft}) - T_A(4 \text{ ft} + 4 \text{ ft}) = 0 \]  

Solving gives

\[ T_A = 20 \text{ lb} \]

8) Tensions in cord on either side of drum

9) Because the cord is on the verge of slipping, we can apply the equation for belt friction:

\[ T_2 = T_1 e^{\mu \beta} \]

where \( T_2 = 20 \text{ lb} \) is the tension in the direction of impending slip and \( T_1 = T_B \) is the tension opposite the direction of impending motion.
10) Geometry

\[ \beta = \pi \]

11) With \( \beta = \pi \) and \( \mu = 0.3 \), Eq. 3 becomes

\[ T_2 = T_1 e^{\mu \beta} \quad \text{(Eq. 3 repeated)} \]

Solving gives

\[ T_B = 7.79 \text{ lb} \]

Eq. 1 then gives

\[ W = T_B = 7.79 \text{ lb} \quad \leftarrow \text{Ans.} \]
5. Determine the largest value $W$ of the weight of block B for which neither block will move. The coefficients of static friction are 0.2 between the blocks and the planes, and 0.25 between the cord and the drum.

```latex
\begin{align*}
\text{Free-body diagram of block A} \\
\text{Impending motion of block A (Since we are to determine the "largest value of the weight" of block B, block B must be about to move down the plane. Thus block A must be about to move \textit{up} the plane.)} \\
\text{Equilibrium equations for block A:} \\
+ \sum F_x &= 0: T_A - f_A - (80 \text{ lb}) \sin \theta \\
\sum F_y &= 0: N_A - (80 \text{ lb}) \cos \theta = 0 \\
\text{Slip impends, so} \\
f_A &= f_{A-max} \\
&= \mu N_A \\
&= (0.2) N_A
\end{align*}
```
Solving Eqs. 1, 2, and 3, with $\theta = 70^\circ$ gives

$T_A = 80.65$ lb

$f_A = 5.47$ lb

$N_A = 27.36$ lb

Because the cord is about to slip over the drum, we can apply the equation for belt friction:

$$T_2 = T_1 e^{\mu \text{drum} \beta}$$

where $T_2 = T_B$ is the tension in the direction of impending slip, and $T_1 = 80.65$ lb is the tension opposite to the direction of impending motion.
Substituting $\beta = \frac{2\pi}{3}$, $T_2 = T_B$, $T_1 = 80.65$ lb, and $\mu_{\text{drum}} = 0.25$ in Eq. 4 gives

$$T_B = (80.65 \text{ lb}) e^{0.25(2\pi/3)}$$

Evaluating the right hand side yields

$$T_B = 136.1 \text{ lb}$$
7.4 Flat Belts Example 5, page 4 of 4

Free-body diagram of block B

Equilibrium equations for block B:

\[ \Sigma F_x' = 0: \quad N_B - W \cos \phi = 0 \quad (5) \]

\[ \Sigma F_y' = 0: \quad 136.1 \text{ lb} + f_B - W \sin \phi = 0 \quad (6) \]

\[ f_B = f_{B\text{-max}} = \mu N_B = (0.2)N_B \quad (7) \]

Solving Eqs. 5, 6, and 7, with \( \phi = 50^\circ \) gives

\[ f_B = 27.4 \text{ lb} \]

\[ N_B = 137.2 \text{ lb} \]

\[ W = 213 \text{ lb} \quad \leftarrow \text{Ans.} \]
6. A motor attached to pulley A drives the pulley clockwise with a 200 lb-in. torque. The flat belt then overcomes the resisting torque $T$ at pulley B and rotates the pulley B clockwise. Determine the minimum tension that can exist in the belt without causing the belt to slip at pulley A. Also determine the corresponding resisting torque $T$. The coefficient of static friction between the belt and the pulleys is 0.3.
1. Free-body diagram of pulley A

2. Impending motion of belt relative to pulley
(Since pulley A is driven by a clockwise torque, the pulley would slip in a clockwise sense relative to the belt. Thus an observer at point D on the pulley would see the belt move in the direction shown.)

3. Equilibrium equation for pulley A

\[ \sum M_A = 0: T_D(4 \text{ in.}) - T_C(4 \text{ in.}) - 200 \text{ lb-in.} = 0 \]  

Because the belt is about to slip on the pulley, the belt friction equation applies:

\[ T_2 = T_1 e^{\mu \beta} \]

where \( T_2 = T_D \), the tension in the direction of impending motion, and \( T_1 = T_C \), so

\[ T_D = T_C e^{\mu \beta} \]
### 7.4 Flat Belts Example 6, page 3 of 4

**4 Geometry**

![Diagram of flat belt setup with geometry labels]

From triangle ABC,

\[
\cos \left( \frac{\beta}{2} \right) = \frac{3 \text{ in.}}{22 \text{ in.}}
\]

Solving gives \( \beta = 164.33^\circ = 2.868 \text{ rad} \).

So Eq. 2, with \( \mu = 0.3 \), becomes

\[
T_D = T_C e^{0.3(2.868)}
\]

**5 Solving Eqs. 1 and 3 simultaneously gives**

- \( T_C = 36.65 \text{ lb} \)  
- \( T_D = 86.65 \text{ lb} \) \( \leftarrow \text{Ans.} \)

**6 Free-body diagram of pulley B**

![Free-body diagram of pulley B]

**7 Equilibrium equation for pulley B**

\[ \sum M_B = 0: \ T + (36.65 \text{ lb})7 \text{ in.} - (86.65 \text{ lb})7 \text{ in.} = 0 \]

Solving gives

\( T = 350 \text{ lb-in.} \) \( \leftarrow \text{Ans.} \)
Finally, we must check that pulley B does not slip. But this follows from the observation that pulley B has a larger angle of wrap than pulley A. Thus it must be able to carry a maximum possible tension larger than the 86.5 lb maximum tension that pulley A carries.

More precisely, because

\[ \beta' > \beta \]

we know that

\[ e^{\mu \beta'} > e^{\mu \beta}. \]

Multiplying through by 36.65 lb gives

\[ \frac{(36.65 \text{ lb})e^{\mu \beta'}}{86.65 \text{ lb}} > \frac{(36.65 \text{ lb})e^{\mu \beta}}{86.65 \text{ lb}}, \text{ by Eq. 2} \]

Thus \( T_{\text{max-B}} > 86.65 \text{ lb} \) and pulley B won't slip.
7.4 Flat Belts Example 7, page 1 of 2

7. If the coefficient of static friction between the fixed drums D and E and the ropes is 0.35, determine the largest weight W that can be supported.

1. Free-body diagram of block C

2. Equilibrium equation for block C

\[ \sum F_y = 0: \ T_E + T_D - W = 0 \] (1)
Rope tensions acting on drum D

Impending motion (Since we are to determine the largest value of weight of block C that can be supported, block C must be about to move down, so the rope connected to C must also be about to move down.)

Because slip is about to occur, the belt-friction equation applies:

\[ T_2 = T_1 e^{\mu \beta} \]

Here, \( T_2 = T_D \), \( T_1 = 100 \text{ lb} \), \( \mu = 0.35 \), and \( \beta = \pi \), so

\[ T_D = (100 \text{ lb}) e^{0.35 \pi} \]

\[ = 300.3 \text{ lb} \]  

Rope tensions acting on drum E (not drum D)

Flat-belt friction equation:

\[ T_2 = T_1 e^{\mu \beta} \]

or,

\[ T_E = (50 \text{ lb}) e^{0.35 \pi} \]

\[ = 150.1 \text{ lb} \]  

Using the results of Eqs. 2 and 3 in Eq. 1 gives

\[ W = 450 \text{ lb} \]  

Ans.
8. Pulley A is rotating under the action of a 6 N-m torque. This motion is transmitted through a flat belt to drive pulley B, which is in turn acted upon by a resisting torque T (the "load" on pulley B). The coefficient of static friction between the belt and the pulleys is 0.45. Determine a) the maximum possible value of T, b) the maximum force in the belt, and c) the corresponding force required in the spring C.
7.4 Flat Belts Example 8, page 2 of 3

1. Free-body diagram of pulley A

2. Impending motion (Since the driving torque acting on pulley A is clockwise, if slip is about to occur, then pulley A will slip clockwise relative to the belt. An observer located on point D on pulley A would see the belt move in the direction shown.)

3. Equilibrium equations for pulley A

\[ \sum F_x = 0: A_x - T_1 - T_2 = 0 \] \hspace{1cm} (1)

\[ \sum F_y = 0: A_y = 0 \] \hspace{1cm} (2)

\[ \sum M_A = 0: T_2(0.08 \text{ m}) - T_1(0.08 \text{ m}) - 6 \text{ N}\cdot\text{m} = 0 \] \hspace{1cm} (3)

4. Flat-belt friction equation:

\[ T_2 = T_1 e^{\mu \beta} \]

\[ = T_1 e^{0.45\pi} \] \hspace{1cm} (4)

Solving Eqs. 1-4 simultaneously gives

\[ A_x = 123.21 \text{ N} \]

\[ A_y = 0 \]

\[ T_1 = 24.11 \text{ N} \]

\[ T_2 = 99.11 \text{ N} \]

\[ \text{Ans.} \]
**7.4 Flat Belts Example 8, page 3 of 3**

5. Free-body diagram of member AC

\[ A_x = 123.21 \text{ N} \]

\[ F_{spring} \]

6. Equilibrium equations for member AC

\[ \sum F_x = 0: F_{spring} - 123.21 \text{ N} = 0 \]  \hspace{1cm} (4)

Solving gives

\[ F_{spring} = 123.21 \text{ N} \] ← Ans.

7. Free-body diagram of pulley B

\[ T_2 = 99.11 \text{ N} \]

\[ T_1 = 24.11 \text{ N} \]

8. Equilibrium equations for pulley B

\[ \sum M_B = 0: T + (24.11 \text{ N})(0.08 \text{ m}) - (99.11 \text{ N})(0.08 \text{ m}) = 0 \]

Solving gives

\[ T = 6 \text{ N} \cdot \text{m} \] ← Ans.

That is, the maximum resisting torque equals the driving torque.