4.5 Equivalent Force-Couple Systems
4.5 Equivalent Force-Couple Systems Example 1, page 1 of 1

1. Replace the force at $A$ by an equivalent force and couple moment at point $O$.

2. Calculate the moment about $O$.

$$M_O = (16 \text{ N})(4 \text{ m}) + (12 \text{ N})(6 \text{ m}) = 136 \text{ N} \cdot \text{m}$$

3. Display the equivalent force and couple moment at point $O$.

1. Express the force in rectangular components.

$$\begin{align*}
(20 \text{ N}) \left( \frac{4}{5} \right) &= 16 \text{ N} \\
(20 \text{ N}) \left( \frac{3}{5} \right) &= 12 \text{ N}
\end{align*}$$

Ans. $136 \text{ N} \cdot \text{m}$
2. The 60-N force acts at point A on the lever as shown. Replace the force at A by a force and couple moment acting at point O that will have an equivalent effect.

1. Calculate the moment about O.

\[ M_O = -(60 \text{ N})(200 \text{ mm}) \cos 60^\circ \]
\[ = -6000 \text{ N} \cdot \text{mm} \]
\[ = -6 \text{ N} \cdot \text{m} \]

2. Display the equivalent force and couple moment at point O.

\[ \text{Ans.} \]
4.5 Equivalent Force-Couple Systems Example 3, page 1 of 1

3. Replace the 2-lb force acting on the end of the bottle opener by an equivalent force and couple moment acting on the underside of the bottle cap at A. Use your results to explain how a bottle opener works.

1. Calculate the couple moment about point A.
\[ M_A = (2 \text{ lb})(3 \text{ in.}) \]
\[ = 6 \text{ lb-in.} \]

2. Display the equivalent force and couple moment at point A.

3. The bottle opener works by pulling (with a 2-lb force) on the edge of the cap while simultaneously twisting (with a 6 lb-in. moment) the entire cap away from the top of the bottle.
4.5 Equivalent Force-Couple Systems Example 4, page 1 of 2

4. Replace the given forces and couple moment by a resultant force and couple moment at A.

![Diagram of forces and couple moment]

1. Express the inclined force in rectangular components.

\[
\begin{align*}
\left( \frac{12}{13} \right) (260 \text{ lb}) &= 240 \text{ lb} \\
\left( \frac{5}{13} \right) (260 \text{ lb}) &= 100 \text{ lb}
\end{align*}
\]

2. Calculate the resultant force

\[
\begin{align*}
\sum F_x &= R_x \\
\rightarrow R_x &= 100 \text{ lb}
\end{align*}
\]

\[
\begin{align*}
\sum F_y &= R_y \\
\uparrow R_y &= 240 \text{ lb} - 120 \text{ lb} \\
&= -360 \text{ lb} \\
\downarrow R_y &= 360 \text{ lb}
\end{align*}
\]

3. Calculate the resultant couple moment about A.

\[
\begin{align*}
\sum M_A &= M_A \\
M_A &= (240 \text{ lb}) (2 \text{ ft} + 4 \text{ ft} + 7 \text{ ft}) + (120 \text{ lb}) (4 \text{ ft} + 7 \text{ ft}) - 800 \text{ lb-ft} \\
&= 3640 \text{ lb-ft}
\end{align*}
\]
4.5 Equivalent Force-Couple Systems Example 4, page 2 of 2

4. Determine the magnitude and direction of the resultant force.

\[ R = \sqrt{(360)^2 + (100)^2} \]

\[ = 374 \text{ lb} \quad \leftarrow \text{Ans.} \]

\[ \theta = \tan^{-1} \frac{360}{100} = 74.5^\circ \quad \leftarrow \text{Ans.} \]

5. Display the equivalent force and couple moment at A.

\[ 374 \text{ lb} \quad 74.5^\circ \]

\[ 3640 \text{ lb}\cdot\text{ft} \quad \leftarrow \text{Ans.} \]
4.5 Equivalent Force-Couple Systems Example 5, page 1 of 3

5. Replace the force $F = 3 \text{ kN}$ acting on corner A of the block by an equivalent force and couple moment acting at the center C of the block.
4.5 Equivalent Force-Couple Systems Example 5, page 2 of 3

Calculate the couple moment about C.

\[ M_C = \mathbf{r}_{CA} \times \mathbf{F} \]

\[ = \{(100 \mathbf{i} - 150 \mathbf{k})\text{mm}\} \times \{-3 \mathbf{j}\}\text{kN} \]

\[ = 100(-3)\mathbf{i} \times \mathbf{j} - 150(-3)\mathbf{k} \times \mathbf{j} \]

\[ = \mathbf{k} = -\mathbf{i} \]

\[ = \{-450 \mathbf{i} - 300 \mathbf{k}\}\text{kN}\cdot\text{mm} \]

\[ = \{-450 \mathbf{i} - 300 \mathbf{k}\}\text{N}\cdot\text{m} \quad \leftarrow \text{Ans.} \]
4.5 Equivalent Force-Couple Systems Example 5, page 3 of 3

2) Display the equivalent force and couple moment at C.

\[ \mathbf{F} = \{-3 \mathbf{j}\} \text{kN} \quad \text{Ans.} \]

\[ \mathbf{M}_C = \{-450 \mathbf{i} - 300 \mathbf{k}\} \text{N·m} \quad \text{Ans.} \]

\[ \mathbf{C} \]

\[ \mathbf{A} \]

\[ 150 \text{ mm} \]

\[ 100 \text{ mm} \]
6. Replace the forces acting on the ice auger by an equivalent force and couple moment acting at A.
Calculate the resultant force

1. \( R_x = \Sigma F_x \): \( R_x = 0 \)
2. \( R_y = \Sigma F_y \): \( R_y = -4 \text{ lb} \)
3. \( R_z = \Sigma F_z \): \( R_z = 7 \text{ lb} \)
Calculate the resultant couple moment about A.

\[ \mathbf{M}_A^R = \sum \mathbf{M}_A \]

\[ = \mathbf{r}_{AC} \times \{-4\mathbf{j}\} \text{ lb} + \mathbf{r}_{AB} \times \{7\mathbf{k}\} \text{ lb} \]

\[ = \mathbf{0}, \text{ because } \mathbf{r}_{AC} \text{ and } \{-4\mathbf{j}\} \text{ are parallel,} \]

or, what amounts to the same thing,

the line of action of the \{-4\mathbf{j}\} \text{ lb force}

passes throughout point A.

\[ = \mathbf{0} + \{-6\mathbf{i} + 48\mathbf{j}\} \text{ in.} \times \{7\mathbf{k}\} \text{ lb} \]

\[ = -6(7)\mathbf{i} \times \mathbf{k} + 48(7)\mathbf{j} \times \mathbf{k} \]

\[ = -7\mathbf{j} = \mathbf{i} \]

\[ = \{336\mathbf{i} + 42\mathbf{j}\} \text{ lb}\cdot\text{in.} \]
**4.5 Equivalent Force-Couple Systems Example 6, page 4 of 4**

\[
\begin{align*}
M_A^R &= \{336i + 42j\} \text{ lb·in.} \\
R &= \{-4j + 7k\} \text{ lb}
\end{align*}
\]

\(\text{Ans.}\)
7. Replace the forces and couple moment by a single force and specify where it acts.

```
A  B  C  D

3 kip  4 kip

20 kip·ft

2 ft  8 ft  4 ft  3 ft

40°
```
4.5 Equivalent Force-Couple Systems Example 7, page 2 of 3

Resolve the inclined force into rectangular components.

\[ (4 \text{ kip}) \cos 40^\circ = 3.064 \text{ kip} \]
\[ (4 \text{ kip}) \sin 40^\circ = 2.571 \text{ kip} \]

\[ \tan^{-1} \frac{3.064}{5.571} = 61.2^\circ \]

Calculate the resultant force

\[ R_x = \Sigma F_x: \quad R_x = -3.064 \text{ kip} = 3.064 \text{ kip} \quad \leftarrow \]
\[ R_y = \Sigma F_y: \quad R_y = -3 \text{ kip} - 2.571 \text{ kip} = 5.571 \text{ kip} \quad \downarrow \]

\[ R = \sqrt{(3.064)^2 + (5.571)^2} = 6.358 \text{ kip} \quad \leftarrow \text{Ans.} \]
4.5 Equivalent Force-Couple Systems Example 7, page 3 of 3

4 This is the original force-couple moment system.

5 This is a new force-couple system that we want to make equivalent to the original force-couple system. We already know that the forces are equivalent because \( R \) is the resultant of the forces in the original system. Now we have to make sure that the moment is also equivalent. We do this by placing \( R \) at some unknown distance \( d \) from the left end and then choosing \( d \) so that the moment of this new system is the same as that of the original system.

6 We equate the moment of the new system, about point A, to the moment of the original force-couple system (The choice of point A for summing moments is arbitrary; any other point would work as well, except that we must use the same point for both the original system and the new system.)

7 \( M_A^R = \Sigma M_A \)

or,

\[ -(5.571 \text{ kip})d = 20 \text{ kip}\cdot\text{ft} - (3 \text{ kip})(2 \text{ ft}) - (2.571 \text{ kip})(2 \text{ ft} + 8 \text{ ft}) \]

Solving gives

\[ d = 2.10 \text{ ft} \]

← Ans.
4.5 Equivalent Force-Couple Systems Example 8, page 1 of 5

8. Replace the forces acting on the frame by a single force and specify where its line of action intersects a) member BCD and b) member AB.

\[ R_x = \sum F_x: \quad R_x = -800 \text{ N} = 800 \text{ N} \leftarrow \]

\[ R_y = \sum F_y: \quad R_y = -400 \text{ N} - 600 \text{ N} - 300 \text{ N} \]

\[ = -1300 \text{ N} \]

\[ = 1300 \text{ N} \downarrow \]

\[ R = \sqrt{(800)^2 + (1300)^2} = 1526 \text{ N} \leftarrow \text{Ans.} \]

\[ \theta = \tan^{-1} \left( \frac{1300}{800} \right) = 58.4^\circ \leftarrow \text{Ans.} \]
Part a) To determine where the line of action of the resultant force intersects member BCD, place the force on BCD, an unknown distance $d$ from point B.

Choose $d$ so that the moment about B of the resultant force equals the moment of the original force system. (The choice of point B was arbitrary.)

$$M_B^R = \sum M_B$$

or

$$-(1300 \text{ N})d = -(600 \text{ N})(4 \text{ m}) - (300 \text{ N})(4 \text{ m} + 4 \text{ m}) - (800 \text{ N})(2 \text{ m})$$

Solving gives

$$d = 4.92 \text{ m} \quad \text{Ans.}$$
Part b) To determine where the line of action of the resultant force intersects member AB, place the force a distance d' from point B.

Choose d' so that the moment about B of the resultant force equals the moment of the original force system.

\[ M_B^R = \Sigma M_B \]

or

\[ -(800 \text{ N})d' = -(600 \text{ N})(4 \text{ m}) - (300 \text{ N})(4 \text{ m} + 4 \text{ m}) - (800 \text{ N})(2 \text{ m}) \]

Solving gives

\[ d' = 8.0 \text{ m} \quad \leftarrow \text{Ans.} \]
The intersection of the line of action with a line drawn through A and B occurs below point B.
4.5 Equivalent Force-Couple Systems Example 8, page 5 of 5

7 Alternative solution for part b. Once we know where the line of action intersects member BCD, we can use geometry to find the intersection with AB.

8 From triangle CBG,

\[ \tan 58.4^\circ = \frac{d'}{4.92} \]

Solving gives,

\[ d' = 8.0 \text{ m} \]

(same result as before)
9. Determine the value of force P such that the line of action of the resultant of the forces acting on the truss passes through the support at H. Also determine the magnitude of the resultant.
Determine the resultant force.

\[ R_x = \sum F_x: \quad R_x = 130 \text{ lb} - P \quad (1) \]

\[ R_y = \sum F_y: \quad R_y = -225.2 \text{ lb} - 160 \text{ lb} - 200 \text{ lb} - 180 \text{ lb} \]

\[ = -765.2 \text{ lb} \]

\[ = 765.2 \text{ lb} \downarrow \quad (2) \]
Choose force $P$ so that the moment about $H$ of the resultant force equals the moment of the original force system about $H$. Any other point besides $H$ could be used, but $H$ has the advantage that the moment of the resultant $R$ is zero, since $R$ is known (as part of the statement of the problem) to pass through $H$.

2. Choose force $P$ so that the moment about $H$ of the resultant force equals the moment of the original force system about $H$. Any other point besides $H$ could be used, but $H$ has the advantage that the moment of the resultant $R$ is zero, since $R$ is known (as part of the statement of the problem) to pass through $H$.

3. \[ M_H^R = \sum M_H : \]
\[ 0 = (225.2 \text{ lb})(6 \text{ ft} + 6 \text{ ft}) - (130 \text{ lb})(8 \text{ ft}) + (160 \text{ lb})(6 \text{ ft}) - (180 \text{ lb})(6 \text{ ft}) + P(8) \]

Solving gives
\[ P = -192.8 \text{ lb} \]
\[ R_x = 130 \text{ lb} - P \]
\[ R_y = 765.2 \text{ lb} \]
\[ R = 322.8 \text{ lb} \]

\[ \leftarrow \text{Ans.} \]
4.5 Equivalent Force-Couple Systems Example 9, page 4 of 4

Magnitude of resultant.

From Eqs. 1 and 2,

\[ R = \sqrt{(322.8)^2 + (-765.2)^2} = 831 \text{ lb} \]

← Ans.
4.5 Equivalent Force-Couple Systems Example 10, page 1 of 4

10. A machine part is loaded as shown. The part is to be attached to a supporting structure by a single bolt. Determine the equation of the line defining possible positions of the bolt for which the given loading would not cause the part to rotate. Also, determine the magnitude and direction of the resultant force.
1. Determine the resultant force.

\[ R_x = \Sigma F_x : \quad R_x = -40 \text{ N} = 40 \text{ N} \leftarrow \]
\[ R_y = \Sigma F_y : \quad R_y = -69.28 \text{ N} - 60 \text{ N} \]
\[ = -129.28 \text{ N} \]
\[ = 129.28 \text{ N} \downarrow \]

\[ R = \sqrt{(40)^2 + (129.28)^2} = 135.33 \text{ N} \leftarrow \text{Ans.} \]

\[ \theta = \tan^{-1} \left( \frac{129.28}{40} \right) = 72.8^\circ \leftarrow \text{Ans.} \]
Rotation is caused by moment. If the given force-couple system is replaced by a single equivalent force (the resultant) and a specified line of action, then the moment would be zero about any point on the line of action. So the line of action is the line on which the bolt should be placed to prevent rotation.

To find the equation of the line of action, place the resultant at a general point \((x, y)\). Then equate moments about, say, point \(O\) for the resultant (the figure on the right) and the original loading (the figure on the left):

\[ M_O^R = \Sigma M_O \]

or

\[
(40 \text{ N})y - (129.28 \text{ N})x = 90 \text{ N}\cdot\text{m} - 20 \text{ N}\cdot\text{m} + (40 \text{ N})(0.3 \text{ m} + 0.5 \text{ m}) - (69.28 \text{ N})(0.4 \text{ m}) - (60 \text{ N})(0.4 \text{ m} + 0.6 \text{ m})
\]
4.5 Equivalent Force-Couple Systems Example 10, page 4 of 4

4. Solving for \( y \) gives the equation of a line

\[
y = 3.232 x + 0.357 \quad \text{Ans.}
\]

This line defines the possible locations of the bolt.

5. All points at the top of the machine part have a \( y \) coordinate of

\[
y = 0.3 \text{ m} + 0.5 \text{ m} = 0.8 \text{ m}.
\]

Substituting \( y = 0.8 \text{ m} \) into the equation of the line for the bolt locations,

\[
y = 3.32 x + 0.357
\]

and solving for \( x \) gives

\[
x = 0.133 \text{ m}
\]
4.5 Equivalent Force-Couple Systems Example 11, page 1 of 4

11. The rectangular foundation mat supports the four column loads shown. Determine the magnitude, direction, and point of application of a single force that would be equivalent to the given system of forces.

Determine the resultant.

\[ \sum F_y: \quad F_y = -30 \text{ kip} - 15 \text{ kip} - 20 \text{ kip} - 11 \text{ kip} \]

\[ = -76 \text{ kip} \]

\[ = 76 \text{ kip} \downarrow \]
First, equate moments about the x axis. We can use either the scalar definition of moment, \( M = Fd \), or the vector product definition. Let's use the scalar definition.

To make the resultant force \( R \) equivalent to the original system of forces, place \( R \) at the point \((x, 0, z)\) and then determine values of \( x \) and \( z \) such that \( R \) produces the same moments about the x and z axes as the given forces produce.

First, equate moments about the x axis. We can use either the scalar definition of moment, \( M = Fd \), or the vector product definition. Let's use the scalar definition.

\[ M_x^R = \Sigma Fd: \quad (76 \text{ kip})z = (20 \text{ kip})(8 \text{ ft} + 3 \text{ ft}) + (15 \text{ kip})(8 \text{ ft} + 3 \text{ ft}) + (11 \text{ kip})(3 \text{ ft}) + (30 \text{ kip})(0) \]
4.5 Equivalent Force-Couple Systems Example 11, page 3 of 4

5) Solving Eq. 1 gives

\[ z = 5.5 \text{ ft} \quad \text{Ans.} \]

Next, equate moments about the z axis.

6) Use two-dimensional views.

7) \[ M_z^R = \sum Fd: \quad -(76 \text{ kip})x = -(20 \text{ kip})(10 \text{ ft}) - (11 \text{ kip})(10 \text{ ft} + 5 \text{ ft}) \]
Solving Eq. 2 gives
\[ x = 4.80 \text{ ft} \quad \leftarrow \text{Ans.} \]
12. Three signs are supported by an arch over a highway and are acted upon by the wind forces shown. Replace the forces by an equivalent force and specify its point of application.

\[ R_z = \sum F_z: \quad R_z = -300 \text{ N} - 850 \text{ N} - 400 \text{ N} \]

\[ = -1550 \text{ N} \]

\[ = -1.55 \text{ kN} \quad \rightarrow \text{Ans.} \]
4.5 Equivalent Force-Couple Systems Example 12, page 2 of 3

2. Place the resultant R at the arbitrary point \((x, y, 0)\) and then determine values of \(x\) and \(y\) such that \(R\) produces the same moments about the \(x\) and \(y\) axes as the given forces produce.

3. First equate moments about the \(x\) axis.

4. \(\mathbf{M}_x^R = \sum \mathbf{F}_d: -(1550 \text{ N})y = - (850 \text{ N})(6 \text{ m}) - (400 \text{ N})(4 \text{ m}) - (300 \text{ N})(3.5 \text{ m}) \) (1)
Solving Eq. 1 gives

\[ y = 5.0 \text{ m} \quad \leftarrow \text{Ans.} \]

Next, equate moments about the y axis.

\[ M_x^R = \sum F_d: (1550 \text{ N})x = (300 \text{ N})(1 \text{ m}) + (850 \text{ N})(1 \text{ m} + 5 \text{ m}) + (400 \text{ N})(1 \text{ m} + 5 \text{ m} + 4 \text{ m}) \]

Solving gives

\[ x = 6.06 \text{ m} \quad \leftarrow \text{Ans.} \]
13. Three forces act on the pipe assembly. Determine the magnitude of forces P and Q if the resultant of all three forces is to act on point A. Also determine the magnitude of the resultant.

Express the resultant R in terms of P and Q.

\[ +\downarrow R = \sum F_y \rightarrow R = P + Q + 200 \text{ N} \]  \hspace{1cm} (1)
4.5 Equivalent Force-Couple Systems Example 13, page 2 of 6

2 Place the resultant R at point A and then determine values of P and Q such that R produces the same moment about the x and z axes as the given forces, P, Q, and 200 N, produce.

3 We can use either the scalar definition of moment $M = Fd$, or the vector definition, $M = r \times F$. Let's use the vector definition and calculate moments with respect to point O.

4 Introduce position vectors, all with tails at point O.

$$r_{OB} = \{3i\} \text{ m}$$

$$r_{OC} = \{5i + 1.5k\} \text{ m}$$

$$r_{OD} = \{5i + 2.5k\} \text{ m}$$

$$r_{OA} = \{3i + 1.5k\} \text{ m}$$

$$R = (-P - Q - 200 \text{ N})j$$
4.5 Equivalent Force-Couple Systems Example 13, page 3 of 6

5) For equivalence, equate the moment of the resultant \( R \) about point \( O \) to the moment of the given forces about \( O \).

\[
M_O^R = \sum M_O: r_{OA} \times R = r_{OB} \times \{-Pj\} + r_{OC} \times \{-Qj\} + r_{OD} \times \{-200j\}
\]

or

\[
(3i + 1.5k) \times (-P - Q - 200)j = \{3i\} \times \{-Pj\} + \{5i + 1.5k\} \times \{-Qj\} + \{5i + 2.5k\} \times \{-200j\}
\]

\[
3(-P - Q - 200)i \times j + 1.5(-P - Q - 200)k \times j = 3(-P)i \times j + 5(-Q)i \times j + 1.5(-Q)k \times j + 5(-200)i \times j + 2.5(-200)k \times j
\]

\[
= k \quad = -i \quad = k \quad = k \quad = -i \quad = k \quad = -i
\]

6) Collecting coefficients of \( i, j, \) and \( k \) gives

\[
1.5(P + Q + 200)i - 3(P + Q + 200)k = (1.5Q + 500)i + (-3P - 5Q - 1000)k
\]

Equating coefficients of \( i \) gives

\[
1.5(P + Q + 200) = 1.5Q + 500 \quad \text{(2)}
\]

Equating coefficients of \( k \) gives

\[
-3(P + Q + 200) = -3P - 5Q - 1000 \quad \text{(3)}
\]
4.5 Equivalent Force-Couple Systems Example 13, page 4 of 6

7) Solving Eqs. 2 and 3 gives,

\[ P = 133.3\text{ N} \]
\[ Q = -200\text{ N} = 200\text{ N} \]

Using these values in Eq. 1 gives

\[ R = P + Q + 200\text{ N} \]
\[ = 133.3\text{ N} \]

8) P and Q were defined as downward directed forces.

\[ R = \begin{align*}
\begin{cases}
P \\
133.3\text{ N}
\end{cases} +
\begin{cases}
Q \\
-200\text{ N}
\end{cases} +
\begin{cases}
200\text{ N}
\end{cases}
\end{align*} \]
\[ = 133.3\text{ N} \]
Alternative solution: the computation are simplified somewhat if we sum moments about point A instead of point O.

Introduce position vectors, all with tails at point A.

10. \[ \mathbf{r}_{AB} = \{-1.5\hat{k}\} \text{ m} \]
\[ \mathbf{r}_{AC} = \{2\hat{i}\} \text{ m} \]
\[ \mathbf{r}_{AD} = \{2\hat{i} + \hat{k}\} \text{ m} \]

11. Equate the moment of R about point A to the moment of the given forces about A.
4.5 Equivalent Force-Couple Systems Example 13, page 6 of 6

Because R passes through point A, the moment is zero.

\[ M_A^R = \sum M_A: 0 = r_{AB} \times \{-Pj\} + r_{AC} \times \{-Qj\} + r_{AD} \times \{-200j\} \]

Equating coefficients of \( i \) gives

\[ 0 = 1.5P + 200 \quad (4) \]

Equating coefficients of \( k \) gives

\[ 0 = 2Q + 400 \quad (5) \]

Solving Eqs. 4 and 5 gives,

\[ P = 133.3 \text{ N} \]
\[ Q = -200 \text{ N} \]

Same result as before.
4.5 Equivalent Force-Couple Systems Example 14, page 1 of 4

14. The end plate of a pressurized tank is held in place by forces from three bolts. Determine the required value of bolt-force $P$ and angle $\theta$ if the resultant of the three bolt forces is to act through the center of the plate at $O$. 

![Diagram showing forces and angles](image-url)
Place the resultant, \( R \), at point \( O \) and then equate the moment of \( R \) about \( O \) (which is zero because \( R \) passes through \( O \)) to the moment of the given forces about \( O \).

1. **Position Vectors**

   - \( \mathbf{r}_{OA} = \{0.5j\} \) m
   - \( \mathbf{r}_{OB} = \{0.5 \cos 40^\circ i - 0.5 \sin 40^\circ j\} \) m = \( \{0.3830i - 0.3214j\} \) m
   - \( \mathbf{r}_{OC} = \{-0.5 \cos \theta i - 0.5 \sin \theta j\} \) m

   ![Diagram showing position vectors](image_url)
4.5 Equivalent Force-Couple Systems Example 14, page 3 of 4

3. Equate moments about O.

\[ \mathbf{M}_O^R = \sum \mathbf{M}_O : \mathbf{0} = \mathbf{r}_{OA} \times \{-700 \mathbf{k}\} \text{ N} + \mathbf{r}_{OB} \times \{-500 \mathbf{k}\} \text{ N} + \mathbf{r}_{OC} \times \{ -P \mathbf{k}\} \text{ N} \]

R passes through O, so produces zero moment.

or

\[ \mathbf{0} = \{0.5 \mathbf{j}\} \times \{-700 \mathbf{k}\} + \{0.3830 \mathbf{i} - 0.3214 \mathbf{j}\} \times \{-500 \mathbf{k}\} + \times \{-0.5 \cos \theta \mathbf{i} - 0.5 \sin \theta \mathbf{j}\} \times \{-P \mathbf{k}\} \]

\[ \mathbf{0} = 0.5(-700) \mathbf{j} \times \mathbf{k} + 0.3830(-500) \mathbf{i} \times \mathbf{k} - 0.3214(-500) \mathbf{j} \times \mathbf{k} - 0.5 \cos \theta (-P) \mathbf{i} \times \mathbf{k} - 0.5 \sin \theta (-P) \mathbf{j} \times \mathbf{k} \]

\[ = \mathbf{i} \quad = -\mathbf{j} \quad = \mathbf{i} \quad = -\mathbf{j} \quad = \mathbf{i} \]

4. Collecting coefficients of \( \mathbf{i} \) and \( \mathbf{j} \) gives

\[ 0 \mathbf{i} + 0 \mathbf{j} = [0.5(-700) - 0.3214(-500) - 0.5 \sin \theta (-P)] \mathbf{i} + [0.3830(500) - 0.5 \cos \theta (P)] \mathbf{j} \]

(1)

Equating coefficients of \( \mathbf{i} \) gives

\[ 0 = [0.5(-700) - 0.3214(-500) - 0.5 \sin \theta (-P)] \]

or, after some arithmetic,

\[ P \sin \theta = 378.6 \]

(2)
4.5 Equivalent Force-Couple Systems Example 14, page 4 of 4

5 Equating coefficients of \( j \) in Eq. 1 gives

\[
0 = 0.3830(500) - 0.5 \cos \theta(P)
\]

or, after some arithmetic,

\[
P \cos \theta = 383.0
\]

(3)

6 Eqs. 2 and 3 are best solved by using a calculator capable of solving simultaneous nonlinear equations. Alternatively, dividing Eq. 2 by Eq. 3 gives

\[
\frac{P \sin \theta}{P \cos \theta} = \frac{378.6}{383.0}
\]

\[
\tan \theta
\]

Solving for \( \theta \) gives,

\[
\theta = 44.7^\circ \quad \text{Ans.}
\]

Substituting for \( \theta \) in Eq. 3 gives,

\[
P \cos \theta = 383.0
\]

\[
44.7^\circ
\]

Solving gives

\[
P = 539 \text{ N} \quad \text{Ans.}
\]
15. Replace the system of forces by a wrench. Determine the pitch and axis of the wrench.
4.5 Equivalent Force-Couple Systems Example 15, page 2 of 6

1. Express the 140-N force in terms of rectangular components. First introduce a position vector \( \mathbf{r}_{AB} \) from A to B.

\[
\mathbf{r}_{AB} = \{4 \mathbf{i} + 12 \mathbf{j} - 6 \mathbf{k}\} \text{ m}
\]

2. Thus the vector form of the 140-N force is

\[
\mathbf{F}_{AB} = (140 \text{ N}) \frac{\mathbf{r}_{AB}}{r_{AB}}
\]

\[
= (140 \text{ N}) \frac{4 \mathbf{i} + 12 \mathbf{j} - 6 \mathbf{k}}{\sqrt{4^2 + 12^2 + (-6)^2}} \approx 140 \text{ N}
\]

\[
= \{40 \mathbf{i} + 120 \mathbf{j} - 60 \mathbf{k}\} \text{ N} \quad (1)
\]

3. The resultant of all three forces is

\[
\mathbf{R} = \sum \mathbf{F}
\]

\[
= \{40 \mathbf{i} + 120 \mathbf{j} - 60 \mathbf{k}\} \text{ N} - \{40 \mathbf{i}\} \text{ N} + \{60 \mathbf{k}\} \text{ N}
\]

\[
= 120 \mathbf{j} \quad (2)
\]

4. Forces along x and z axes
Moment arm is zero.
(40 N force passes through O)

Moment resultant about O:
\[ M_O^R = r_{OA} \times F_{AB} + r_{OA} \times (60k) + 0 \times (-40i) \]

\[ = (6k) \times (40i + 120j - 60k) + (6k) \times (60k) \]

\[ = 6(40)k \times i + 6(120)k \times j + 6(-60)k \times k + 6(60)k \times k \]

\[ = 6j - 6i = 0 = 0 \]

\[ = [-720i + 240j] \text{ N.m} \]
4.5 Equivalent Force-Couple Systems Example 15, page 4 of 6

7) Express \( \mathbf{M}_o^R \) in terms of components parallel \( \mathbf{M}_\parallel \) and perpendicular \( \mathbf{M}_\perp \) to \( \mathbf{R} \).

\[
\mathbf{M}_o^R = \{-720i + 240j\} \text{ N\cdot m}
\]

\[
\mathbf{M}_\perp = \{-720i\} \text{ N\cdot m}
\]

\[
\mathbf{M}_\parallel = \{240j\} \text{ N\cdot m}
\]

\[
\mathbf{R} = \{120j\} \text{ N}
\]

By definition, pitch of wrench, \( p = \frac{\mathbf{M}_\parallel}{\mathbf{R}} = \frac{240}{120} = 2 \text{ m} \) ← Ans.
4.5 Equivalent Force-Couple Systems Example 15, page 5 of 6

Now move \( \mathbf{R} \) to a new line of action such that in the new location \( \mathbf{R} \) will produce a moment (about \( O \)) equal to \( M_\perp \).

Equate the moment of \( \mathbf{R} \) in the new position to \( M_\perp \)

\[
\mathbf{r}_{OP} \times \mathbf{R} = M_\perp
\]

or

\[
\{x \mathbf{i} + z \mathbf{k}\} \times \{120 \mathbf{j}\} = -720 \mathbf{i}
\]

\[
x(120) \mathbf{i} \times \mathbf{j} + z(120) \mathbf{k} \times \mathbf{j} = -720 \mathbf{i}
\]

\[
= \mathbf{k} = -\mathbf{i}
\]

\[-(120z) \mathbf{i} + (120x) \mathbf{k} = -720 \mathbf{i} + 0 \mathbf{k}
\]

Equating coefficients of \( \mathbf{i} \) gives

\[-120z = -720
\]

\[z = \frac{-720}{-120} = 6 \text{ m}
\]

Equating coefficients of \( \mathbf{k} \) gives

\[120x = 0
\]

\[x = 0
\]
4.5 Equivalent Force-Couple Systems Example 15, page 6 of 6

11. The axis of the wrench is a vertical line (same direction as \( \mathbf{R} = \{120j\} \)) passing through the point (0, 0, 6 m)

\[
\begin{align*}
\mathbf{R} &= \{120j\} \text{ N} \\
\mathbf{M} &= \{240j\} \text{ N m}
\end{align*}
\]

\( \leftarrow \text{Ans.} \)

12. Because a couple moment has the same value about all points it can be moved to the new line of action of \( \mathbf{R} \) to form the wrench.

\( \leftarrow \text{Ans.} \)
4.5 Equivalent Force-Couple Systems Example 16, page 1 of 14

16. In a machining operation, holes are simultaneously drilled at points A and B of the wedge. The drill at A produces a force and couple moment perpendicular to the planar surface at A. The force and couple moment at B are similarly perpendicular to the planar surface at B. Replace the forces and couple moments by a wrench. Determine

a) the magnitude $R$ of the resultant force,
b) the pitch of the wrench,
c) the axis of the wrench, and
d) the point where the axis intersects the $x$-$z$ plane.
4.5 Equivalent Force-Couple Systems Example 16, page 2 of 14

1. Express the forces and couple moments in rectangular components.

2. Geometry

\[ M_1 = -(60 \text{ lb-in.}) \sin 60^\circ \textbf{j} - (60 \text{ lb-in.}) \cos 60^\circ \textbf{k} \]
\[ = \{-51.962 \textbf{j} - 30 \textbf{k}\} \text{ lb-in.} \]  

\[ F_1 = -(4 \text{ lb}) \sin 60^\circ \textbf{j} - (4 \text{ lb}) \cos 60^\circ \textbf{k} \]
\[ = \{-3.464 \textbf{j} - 2 \textbf{k}\} \text{ lb} \]  

View from positive x axis
4.5 Equivalent Force-Couple Systems Example 16, page 3 of 14

\[ M_2 = 80 \text{ lb-in.} \]

\[ F_2 = 6 \text{ lb} \]

\[ F_2 = \{ -6i \} \text{ lb} \quad (3) \]

\[ M_2 = \{ -80i \} \text{ lb-in.} \quad (4) \]
Next, determine the coordinates of points A and B.

\[ A_y = (3 \text{ in.}) \sin 30^\circ = 1.5 \text{ in.} \]
\[ A_z = (6 \text{ in.} + 1 \text{ in.}) - 2.598 \text{ in.} = 4.402 \text{ in.} \]

\[ B_y = 1.5 \text{ in.} \]
\[ B_z = 1 \text{ in.} \]

\[ A_x = 4 \text{ in.} \]
\[ B_x = 4 \text{ in.} + 4 \text{ in.} = 8 \text{ in.} \]

\[ (3 \text{ in.}) \cos 30^\circ = 2.598 \text{ in.} \]
4.5 Equivalent Force-Couple Systems Example 16, page 5 of 14

(9) Introduce position vectors from O, the origin of coordinates, to A and B.

\[ \mathbf{r}_{OA} = A_x \mathbf{i} + A_y \mathbf{j} + A_z \mathbf{k} \]
\[ = \{4\mathbf{i} + 1.5\mathbf{j} + 4.402\mathbf{k}\} \text{ in.} \quad (5) \]

\[ \mathbf{r}_{OB} = B_x \mathbf{i} + B_y \mathbf{j} + B_z \mathbf{k} \]
\[ = \{8\mathbf{i} + 1.5\mathbf{j} + \mathbf{k}\} \text{ in.} \quad (6) \]
4.5 Equivalent Force-Couple Systems Example 16, page 6 of 14

12 Resultant force

\[ \mathbf{R} = \sum \mathbf{F}: \mathbf{R} = \{-3.464\mathbf{j} - 2\mathbf{k}\} + \{-6\mathbf{i}\} \]

\[ = \{-6\mathbf{i} - 3.464\mathbf{j} - 2\mathbf{k}\} \text{ lb} \quad (7) \]

13 Resultant moment about point O

\[ \mathbf{M}_O^R = \sum \mathbf{M}_O^R: \mathbf{M}_O^R = \mathbf{r}_{OA} \times \mathbf{F}_1 + \mathbf{r}_{OB} \times \mathbf{F}_2 \]

\[
\begin{array}{ccc}
\mathbf{M}_O^R = & \mathbf{i} & \mathbf{j} & \mathbf{k} \\
& 4 & 1.5 & 4.402 \\
& 0 & -3.464 & -2 \\
\end{array} + \begin{array}{ccc}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
& 8 & 1.5 & 1 \\
& -6 & 0 & 0 \\
\end{array}
\]

Carrying out the cross products and simplifying gives

\[ \mathbf{M}_O^R = \{12.249\mathbf{i} + 2\mathbf{j} - 4.856\mathbf{k}\} \quad (8) \]
4.5 Equivalent Force-Couple Systems Example 16, page 7 of 14

Display \( \mathbf{R} \) and \( \mathbf{M}_O \) with respect to the \( xyz \) axes.

\[
\mathbf{R} = \{-6i - 3.464j - 2k\} \text{ lb}
\]

\[
\mathbf{M}_O = \{12.249i + 2j - 4.856k\}
\]
Two intersecting vectors define a plane. Display the plane defined by \( \mathbf{R} \) and \( \mathbf{M}_O^R \).
4.5 Equivalent Force-Couple Systems Example 16, page 9 of 14

Express $\mathbf{M}_O^R$ in terms of components parallel $\mathbf{M}_p$ and perpendicular $\mathbf{M}_p$ to $\mathbf{R}$.

In vector form, since $\mathbf{M}_p$ lies in the direction of the unit vector $\mathbf{u}$,

\[ \mathbf{M}_p = \mathbf{M}_p \mathbf{u} \]
\[ = (-9.806) \{-0.832 \mathbf{i} - 0.480 \mathbf{j} - 0.277 \mathbf{k}\} \]
\[ = \{8.159 \mathbf{i} + 4.707 \mathbf{j} + 2.716 \mathbf{k}\} \text{ lb-in.} \] (12)

Unit vector in $\mathbf{R}$ direction

By Eq. 7 for $\mathbf{R}$,

\[ \mathbf{R} = \sqrt{(-6)^2 + (-3.464)^2 + (-2)^2} \]
\[ = 7.211 \text{ lb} \quad \text{Ans.} \] (9)

Thus

\[ \mathbf{u} = \frac{\mathbf{R}}{\mathbf{R}} = \frac{-6 \mathbf{i} - 3.464 \mathbf{j} - 2 \mathbf{k}}{7.211} \]
\[ = -0.832 \mathbf{i} - 0.480 \mathbf{j} - 0.277 \mathbf{k} \] (10)

$\mathbf{M}_p$ = component of $\mathbf{M}_O^R$ in direction of $\mathbf{u}$

\[ = \mathbf{M}_O^R \cdot \mathbf{u} \]
\[ = \{12.249 \mathbf{i} + 2 \mathbf{j} - 4.856 \mathbf{k}\} \cdot \{-0.832 \mathbf{i} - 0.480 \mathbf{j} - 0.277 \mathbf{k}\} \]

Performing the multiplications and then simplifying gives

\[ \mathbf{M}_p = -9.806 \text{ lb-in.} \] (11)
The component of $M^R_0$ perpendicular to $R$ can now be computed by starting with the vector sum

\[ \mathbf{M}^R_0 = \mathbf{M} \perp + \mathbf{M} \parallel \]

and rearranging to get

\[ \mathbf{M} \perp = \mathbf{M}^R_0 - \mathbf{M} \parallel \]

by Eq.8 by Eq.12

\[ = \{12.249i + 2j - 4.856k\} - \{8.159i + 4.707j + 2.716k\} \]

\[ = \{4.090i - 2.707j - 7.572k\} \text{ lb-in.} \] (13)
Now we move the force \( \mathbf{R} \) to a new line of action such that \( \mathbf{R} \) will produce a moment about \( O \) exactly equal to \( M_{\perp} \).
4.5 Equivalent Force-Couple Systems Example 16, page 12 of 14

22) View in terms of xyz axes

23) Because the force $\mathbf{R}$, in its new position, is to create a moment about $O$ equal to $\mathbf{M}_\perp$, we can write

$$\mathbf{r}_{OP} \times \mathbf{R} = \mathbf{M}_\perp$$

by Eq. 13

or,

by Eq. 7

$$\{xi + zk\} \times \{ -6i - 3.464j - 2k \} = 4.090i - 2.707j - 7.572k$$

Performing the multiplications and simplifying gives

$$3.464zi + (2x - 6z)j - 3.464xk = 4.090i - 2.707j - 7.572k$$
4.5 Equivalent Force-Couple Systems Example 16, page 13 of 14

(24) Equating coefficients of \( i \) gives

\[
3.464z = 4.090 \quad (14)
\]

Similarly for \( j \) and \( k \)

\[
2x - 6z = -2.707 \quad (15)
\]

\[
-3.464x = -7.572 \quad (16)
\]

These are three equations in only two unknowns, but the equations are not all independent. Eqs 14 and 16 imply

\[
z = \frac{4.090}{3.464} = 1.181 \quad (17)
\]

\[
x = \frac{-7.572}{-3.464} = 2.186 \quad (18)
\]

Substituting these values into the left hand side of Eq.15 gives

\[
2x - 6z = 2(2.186) - 6(1.181) = -2.714
\]

Thus Eq.15 is satisfied (Round-off, error leads to 2.714 rather than 2.707).
Summary: The effect of the drilling operations on the two surfaces of the wedge is to push the wedge down the wrench axis (direction of $\mathbf{u}$) while simultaneously causing the wedge to tend to rotate counterclockwise (as viewed from a position above the $x$-$z$ plane) about the wrench axis.

25. Because a couple moment is the same about all points, we can move $\mathbf{M}_{||}$ to point $P$. Similarly we can slide $\mathbf{R}$ to $P$ along $\mathbf{R}$'s line of action, by the principle of transmissibility.

26. The axis of the wrench is a line passing through the point (2.186 in., 0, 1.181 in.) with direction, by Eq. 10,

$$\mathbf{u} = -0.832\mathbf{i} - 0.480\mathbf{j} - 0.277\mathbf{k}$$

\[\begin{align*}
\text{Ans.} & \quad \text{Ans.} \\
\mathbf{M} & = \mathbf{M}_{||} \mathbf{u} \\
& = (-9.805 \text{ lb-in.})\mathbf{u} \\
\text{Pitch of wrench} & = \frac{\mathbf{M}_{||}}{\mathbf{R}} \\
& = \frac{-9.806 \text{ lb-in.}}{7.211 \text{ lb}} \\
& = -1.360 \text{ in.} \quad \text{Ans.}
\end{align*}\]

27. Pitch of wrench $= \frac{\mathbf{M}_{||}}{\mathbf{R}}$

28. Summary: The effect of the drilling operations on the two surfaces of the wedge is to push the wedge down the wrench axis (direction of $\mathbf{u}$) while simultaneously causing the wedge to tend to rotate counterclockwise (as viewed from a position above the $x$-$z$ plane) about the wrench axis.