4.2 Moments in Three-Dimensional Force Systems
4.2 Moments in Three-Dimensional Force Systems Procedures and Strategies

Procedures and Strategies for Solving Problems Involving Moments in Three-Dimensional Force Systems

1. To calculate the moment of a force \( \mathbf{F} \) about a point \( O \),
   a) Express \( \mathbf{F} \) in rectangular component form.

   b) Define a position vector \( \mathbf{r} \), with tail at point \( O \) and head at *any point along the line of action* of \( \mathbf{F} \). If you have more than one possible choice for \( \mathbf{r} \), choose the one that gives the simplest form for \( \mathbf{r} \). Express \( \mathbf{r} \) in rectangular component form.

   c) Evaluate the cross product: \( \mathbf{M}_O = \mathbf{r} \times \mathbf{F} \) (If you have a scientific graphing calculator, use its built-in cross-product function).

2. To determine the shortest distance \( d \) between a point \( A \) and a given line, assume that a force of unknown magnitude \( \mathbf{F} \) acts along the line, and then make use of the fact that two different formulas exist for calculating the *magnitude* of the moment \( \mathbf{M}_A \) about \( A \):

   \[
   \mathbf{M}_A = Fd \quad (1)
   \]

   and

   \[
   \mathbf{M}_A = |\mathbf{r} \times \mathbf{F}| \quad (2)
   \]

   where \( \mathbf{r} \) is a positive vector with tail at \( A \) and head on the line. Then

   a) find a unit vector \( \mathbf{u} \) parallel to the line, and express it in rectangular component form;

   b) express the force as \( \mathbf{F} = \mathbf{F} \mathbf{u} \).
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c) express \( \mathbf{r} \) in rectangular component form,

d) compute the cross product

\[
\mathbf{r} \times \mathbf{F} = \mathbf{r} \times (\mathbf{F} \mathbf{u}) \\
= \mathbf{F}(\mathbf{r} \times \mathbf{u})
\]

Then Eqs. 1 and 2 give

\[
\mathbf{F}d = \mathbf{F}|\mathbf{r} \times \mathbf{u}|
\]

Cancel \( \mathbf{F} \).

Thus to calculate the shortest distance \( d \) from point A to a line, just calculate the magnitude of the cross product of \( \mathbf{r} \) and \( \mathbf{u} \).
4.2 Moments in Three-Dimensional Force Systems Problem Statement for Example 1

1. Use the cross-product definition of the moment of a force to determine the moment of the force about point A. Also, compare the sign of the result with that obtained from the scalar definition of positive moment, \( M = Fd \).
4.2 Moments in Three-Dimensional Force Systems Problem Statement for Example 2

2. A force $F = 20$ N is applied to the end of a string of length $L$. The other end of the string is tied to the handle of a wrench as shown. Use the cross-product definition of the moment to determine the moment of $F$ about point $A$. Discuss the effect of distance $L$ on your answer.
4.2 Moments in Three-Dimensional Force Systems Problem Statement for Example 3

3. A shower/bathtub grab bar is being pulled by a force $F = 30$ lb as shown. Determine the moment of $F$ about the support A. Also determine the coordinate direction angles of the moment vector and interpret the result.

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Diagram:
- A shower/bathtub grab bar is shown with a force $F = 30$ lb applied at an angle of $40^\circ$ and $60^\circ$.
- The grab bar is supported at points A and B, with distances of 16 in. and 8 in. respectively.
- The grab bar is shown with dimensions of 5 in. at the top and 3 in. at the bottom.
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4. A force $F = 15 \text{ N}$ acting parallel to the $z$ axis is applied to the handle of a socket wrench to turn a bolt at $A$. Determine the moment of the force about the point $A$. Also, state which component of the moment tends to turn the bolt.

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Diagram:
- A force $F = 15 \text{ N}$ is applied parallel to the $z$ axis.
- The wrench handle is $100 \text{ mm}$ from the bolt at $A$.
- The wrench is $80 \text{ mm}$ from the point $B$.
- The $z$-axis and $y$-axis are marked.
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5. Pulley B is used to drive pulley C. Determine the resultant moment about bearing A produced by the belt forces acting on pulley B. Also, interpret your result.
6. A child on a bicycle collides with a mailbox and exerts the force $F$ shown. If the base of the pole at $O$ will fail if the magnitude of the moment there exceeds 60 N·m, determine if the mailbox will fall over.

\[
F = \{80i + 12j - 10k\} \text{ N}
\]
4.2 Moments in Three-Dimensional Force Systems Problem Statement for Example 7

7. Copper tubing emerges from the wall at A and is subjected to a force F at its free end B. The tubing will fail if the magnitude of the moment at A exceeds 3 N·m. Determine the largest value of the force F that can be applied to the free end of the tubing.
8. Two forces, \( P = 60 \text{ N} \) and \( Q = 80 \text{ N} \) act on the vertices of a cube as shown. Determine the moment of each force about point O, if the length of each edge of the cube is 2 m. Also, determine the shortest distance from O to the line BF.
4.2 Moments in Three-Dimensional Force Systems Problem Statement for Example 9

9. Determine the moment about the screw at A of the force $F = 2 \text{ N}$ applied to the sheet-metal bracket shown. Also, determine the shortest distance from A to the line connecting B and C.
4.2 Moments in Three-Dimensional Force Systems Problem Statement for Example 10

10. If the tension in the cable BC is $T = 80$ lb, determine the moment about point A of the cable force acting on the frame at point B. Also, determine the shortest distance from A to the line through B and C.