2.2 Rectangular Components in Two-Dimensional Force Systems
2.2 Rectangular Components in Two-Dimensional Force Systems Example 1, page 1 of 3

1. Express the 5-kN force in terms of x and y components.

\[ \mathbf{F} = 5 \text{ kN} \]

Because the x and y axes are perpendicular, the parallelogram is a special case—a rectangle.

2. Analyze the triangle forming the lower half of the rectangle.

3. Construct a parallelogram with the 5-kN force as the diagonal and with sides in the x and y directions.
2.2 Rectangular Components in Two-Dimensional Force Systems Example 1, page 2 of 3

4) Calculate $F_x$ from the definition of the cosine:

\[ \cos \theta = \frac{A}{C} \]

so

\[ A = C \times \cos \theta \]

In words,

side adjacent (to angle) = hypotenuse times cosine of angle.

(Memorize this—you will use this relation many times in a course in statics; you don't want to have to think it out each time)

Applying this equation to the force triangle gives:

\[ F_x = (5 \text{ kN}) \times \cos 30^\circ \]

\[ = 4.33 \text{ kN} \quad (1) \]

5) Similarly calculate $F_y$ from the definition of the sine:

\[ \sin \theta = \frac{B}{C} \]

so

\[ B = C \times \sin \theta \]

In words,

side opposite (to angle) = hypotenuse times sine of angle.

(Memorize this.)

Applying this equation to the force triangle gives:

\[ F_y = (5 \text{ kN}) \times \sin 30^\circ \]

\[ = 2.50 \text{ kN} \quad (2) \]
Thus we have resolved the 5-kN force into x and y components.

\[
\begin{align*}
F &= 5 \text{ kN} \\
F_x &= 4.33 \text{ kN} \\
F_y &= 2.50 \text{ kN}
\end{align*}
\]

In terms of base vectors, the force is

\[
F = \{-4.33i + 2.50j\} \text{ kN} \quad \leftarrow \text{Ans.}
\]

The minus sign indicates that the x component points in the negative x direction.
2.2 Rectangular Components in Two-Dimensional Force Systems Example 2, page 1 of 4

2. Resolve the 20-lb force into x and y components.

1. Construct a parallelogram (rectangle) with the 20-lb force as the diagonal.

2. Analyze the triangle forming the lower half of the rectangle.

Equal angles
2.2 Rectangular Components in Two-Dimensional Force Systems Example 2, page 2 of 4

3 Side adjacent = hypotenuse × cos θ

or

\[ F_x = (20 \text{ lb}) \times \cos \theta \]  

Similarly, side opposite = hypotenuse × sin θ

or

\[ F_y = (20 \text{ lb}) \times \sin \theta \]  

4 From the "slope triangle," we see

\[ \cos \theta = \frac{4}{5} \]  
\[ \sin \theta = \frac{3}{5} \]

Note that we do not have to calculate \( \theta \); we already have what we need, \( \sin \theta \) and \( \cos \theta \).
Using Eq. 3 in Eq. 1 gives

\[ F_x = (20 \text{ lb}) \times \left( \frac{4}{5} \right) \]

\[ = 16 \text{ lb} \quad (5) \]

In general, then, get the horizontal force component by multiplying the force by the horizontal side of the slope triangle divided by the hypotenuse (Memorize this result; it is used frequently).
Similarly, using Eq. 4 in Eq. 2 gives
\[ F_y = (20 \text{ lb}) \times \left( \frac{3}{5} \right) \]
\[ = 12 \text{ lb} \quad (6) \]

In general, get the \textit{vertical} force component by multiplying the force by the \textit{vertical} side of the slope triangle divided by the hypotenuse (Memorize this result).

Eqs. 5 and 6 now give the components in terms of base vectors as
\[ \mathbf{F} = \{16i + 12j\} \text{ lb} \quad \leftrightarrow \text{Ans.} \]
2.2 Rectangular Components in Two-Dimensional Force Systems Example 3, page 1 of 2

3. Express the 260-N force in terms of components parallel and perpendicular to the inclined plane.

1. Introduce an inclined x and y coordinate system.

2. Construct a parallelogram (rectangle) with the 260-N force as a diagonal.
2.2 Rectangular Components in Two-Dimensional Force Systems Example 3, page 2 of 2

3 Analyze the triangle forming the lower half of the rectangle.

\[ \sqrt{5^2 + 12^2} = 13 \]

\[ F = 260 \text{ N} \]

4 \[ F_x = (260 \text{ N})(\frac{5}{13}) = 100 \text{ N} \]

5 \[ F_y = (260 \text{ N})(\frac{12}{13}) = 240 \text{ N} \]

6 Representation in terms of components:

\[ \mathbf{F} = \{-100\mathbf{i} - 240\mathbf{j}\} \text{ N} \quad \leftarrow \text{Ans.} \]

7 \( F_x \) points in the negative x direction, and \( F_y \) points in the negative y direction.
2.2 Rectangular Components in Two-Dimensional Force Systems Example 4, page 1 of 3

4. Determine the components of the 160-N force perpendicular and parallel to the axis of the nail.

Introduce an inclined x and y coordinate system.

F = 160 N
20°
15°
2.2 Rectangular Components in Two-Dimensional Force Systems Example 4, page 2 of 3

(2) Geometry

(4) Total angle
= 20° + 15°
= 35°

(3) Equal angles

(5) Draw a parallelogram (rectangle) with the 160-N force as a diagonal.
6) Analyze the triangle forming the bottom half of the rectangle.

\[ F_{y} = (160 \text{ N}) \times \sin 35^\circ = 91.8 \text{ N} \]

\[ F_{x} = (160 \text{ N}) \times \cos 35^\circ = 131.1 \text{ N} \]

7) In terms of base vectors,

\[ \mathbf{F} = \{ 131.1\mathbf{i} - 91.8\mathbf{j} \} \text{ N} \]

← Ans.
5. The connecting rod AB exerts a 2-kN force on the crankshaft at B. Resolve this force into components acting perpendicular to BC and along BC.
2.2 Rectangular Components in Two-Dimensional Force Systems Example 5, page 2 of 3

1) Introduce an inclined x and y coordinate system.

2) Calculate angles

3) Equal

4) Equal

5) Calculate the sum: $20^\circ + 30^\circ = 50^\circ$

6) Calculate components

(2 kN) $\sin 50^\circ = 1.532$ kN

(2 kN) $\cos 50^\circ = 1.286$ kN
2.2 Rectangular Components in Two-Dimensional Force Systems Example 5, page 3 of 3

7) In terms of base vectors,

\[ \mathbf{F} = \{1.532i - 1.286j\} \text{ kN} \]

← Ans.
2.2 Rectangular Components in Two-Dimensional Force Systems Example 6, page 1 of 3

6. Guy wire AB exerts a horizontal component of force of 0.5 kN on the utility pole. Determine the total force from the wire acting on the point of attachment, A. Assume that the force is directed along the wire from A to B.
Express the guy-wire force $F$ in terms of rectangular components.

The horizontal component of force is known to be 0.5 kN.
2.2 Rectangular Components in Two-Dimensional Force Systems Example 6, page 3 of 3

3. Relate $F_x$ to $F$ through geometry.

4. \[ \theta = \tan^{-1} \frac{10 \text{ m}}{5 \text{ m}} = 63.43^\circ \]

5. 0.5 kN = $F \cos \theta$

Solving gives

\[ F = 1.118 \text{ kN} \] ← Ans.
2.2 Rectangular Components in Two-Dimensional Force Systems Example 7, page 1 of 1

7. If the vertical component of the force \( F \) applied to the ring is 10 lb, determine the magnitude \( F \) and also the horizontal component.

1. Express \( F \) in terms of rectangular components.

\[
\begin{align*}
F_x &= F \cos 30^\circ \\
&= (20 \text{ lb}) \cos 30^\circ \\
&= 17.32 \text{ lb} \\
\text{Ans.}
\end{align*}
\]

2. Relate \( F \) to \( F_y \).

\[
\begin{align*}
10 \text{ lb} &= F \sin 30^\circ \\
F &= 20 \text{ lb} \quad \leftarrow \text{Ans.}
\end{align*}
\]

3. Relate \( F_x \) to \( F \).

\[
\begin{align*}
F_x &= F \cos 30^\circ \\
&= (20 \text{ lb}) \cos 30^\circ \\
&= 17.32 \text{ lb} \quad \leftarrow \text{Ans.}
\end{align*}
\]
2.2 Rectangular Components in Two-Dimensional Force Systems Example 8, page 1 of 2

8. The weight W is supported by the boom AB and cable AC. Knowing that the horizontal and vertical components of the cable force at A are 5 kN and 3 kN as shown, determine the distance d.

1. Calculate the angle between $F_{cable}$ and its horizontal component.

\[
\theta = \tan^{-1} \left( \frac{3 \text{ kN}}{5 \text{ kN}} \right) = 30.96^\circ
\]
2.2 Rectangular Components in Two-Dimensional Force Systems Example 8, page 2 of 2

2) Use $\theta$ to calculate $d$.

\[
d = (10 \text{ m}) \tan 30.96^\circ
\]

\[
= 6.0 \text{ m} \quad \text{Ans.}
\]

3) The same result could also have been obtained by using similar triangles.

\[
\frac{3 \text{ kN}}{5 \text{ kN}} = \frac{d}{10 \text{ m}}
\]

Therefore,

\[
d = 6.0 \text{ m} \quad \text{(same as before)}
\]
2.2 Rectangular Components in Two-Dimensional Force Systems Example 9, page 1 of 2

9. Determine the magnitude and direction of the resultant force acting on the hook.

1. Express the forces in x and y components.

\[(20 \text{ lb}) \sin 35^\circ = 11.47 \text{ lb}\]
\[(20 \text{ lb}) \cos 35^\circ = 16.38 \text{ lb}\]
\[(104 \text{ lb}) \left(\frac{5}{13}\right) = 40 \text{ lb}\]
\[(104 \text{ lb}) \left(\frac{12}{13}\right) = 96 \text{ lb}\]
\[\sqrt{5^2 + 12^2} = 13\]

2. Calculate the x and y components of the resultant \( \mathbf{R} \) by summing the components of the given forces algebraically.

\[\begin{align*}
\rightarrow R_x &= \sum F_x : R_x = 16.38 \text{ lb} + 96 \text{ lb} = 112.38 \text{ lb} \\
\downarrow R_y &= \sum F_y : R_y = 11.47 \text{ lb} - 40 \text{ lb} = -28.53 \text{ lb} = 28.53 \text{ lb} \\
\end{align*}\]

(arrow indicates negative y direction)
2.2 Rectangular Components in Two-Dimensional Force Systems Example 9, page 2 of 2

3 Calculate the magnitude and direction of the resultant \( R \).

\[
R = \sqrt{(112.38 \text{ lb})^2 + (28.53 \text{ lb})^2} = 115.9 \text{ lb}
\]

\[
\theta = \tan^{-1} \left( \frac{28.53 \text{ lb}}{112.38 \text{ lb}} \right) = 14.2^\circ
\]

\[\text{Ans.} \] 112.38 lb

\[ \theta \]

\[ \text{Ans.} \] 28.53 lb

\[ R \]

\[ 115.9 \text{ lb} \]
2.2 Rectangular Components in Two-Dimensional Force Systems Example 10, page 1 of 2

10. Determine the magnitude and direction of the resultant force acting on the beam.

1. Resolve the forces into x and y components.

2. Calculate the x and y components of the resultant by summing the components of the given forces algebraically.

\[ R_x = \sum F_x: \quad R_x = 6.128 \text{ kN} - 12 \text{ kN} = -5.872 \text{ kN} = 5.872 \text{ kN} \leftarrow \]

\[ R_y = \sum F_y: \quad R_y = -5.142 \text{ kN} - 9 \text{ kN} + 11 \text{ kN} = -3.142 \text{ kN} = 3.142 \text{ kN} \downarrow \]
2.2 Rectangular Components in Two-Dimensional Force Systems Example 10, page 2 of 2

3. Calculate the magnitude and direction of the resultant.

\[ R = \sqrt{(5.872 \, \text{kN})^2 + (3.142 \, \text{kN})^2} \]

\[ = 6.66 \, \text{kN} \]

\[ \theta = \tan^{-1} \left( \frac{3.142 \, \text{kN}}{5.872 \, \text{kN}} \right) \]

\[ = 28.2^\circ \]

\[ \rightarrow \text{Ans.} \]
11. Determine the magnitude and direction of the resultant force acting on the particle.

![Diagram of forces acting on a particle with coordinates and forces labeled.](image-url)
2.2 Rectangular Components in Two-Dimensional Force Systems Example 11, page 2 of 5

1. We want to compute the x and y components of each force. To do that, we first must compute some angles.

\[ \theta = \tan^{-1} \frac{3 \text{ m}}{6 \text{ m}} = 26.57^\circ \]

Angle for 80-N force.

Components of 80-N force.

\[ (80 \text{ N}) \sin 26.57^\circ = 35.78 \text{ N} \]
\[ (80 \text{ N}) \cos 26.57^\circ = 71.55 \text{ N} \]
Angle and components for 50-N force.

\[ \theta = \tan^{-1} \left( \frac{3 \text{ m}}{5 \text{ m}} \right) = 30.96° \]

\[
(50 \text{ N}) \cos 30.96° = 42.88 \text{ N}
(50 \text{ N}) \sin 30.96° = 25.72 \text{ N}
\]
Angle and components for 25-N force.

\[ \theta = \tan^{-1} \frac{2 \text{ m}}{6 \text{ m}} = 18.43^\circ \]

\[ (25 \text{ N}) \cos 18.43^\circ = 23.72 \text{ N} \]

\[ (25 \text{ N}) \sin 18.43^\circ = 7.90 \text{ N} \]
2.2 Rectangular Components in Two-Dimensional Force Systems Example 11, page 5 of 5

4 Sum the components algebraically.

\[ R_x = \sum F_x: \quad R_x = 35.78 \text{ N} - 42.88 \text{ N} + 23.72 = 16.62 \text{ N} \rightarrow \]
\[ R_y = \sum F_y: \quad R_y = 71.55 \text{ N} - 25.72 \text{ N} - 7.90 \text{ N} = 37.93 \text{ N} \uparrow \]

5 Calculate the magnitude and direction of the resultant \( R \).

\[ R = \sqrt{(37.93 \text{ N})^2 + (16.62 \text{ N})^2} \]
\[ = 41.4 \text{ N} \]
\[ \theta = \tan^{-1} \frac{37.93 \text{ N}}{16.62 \text{ N}} \]
\[ = 66.3^\circ \]

\[ \text{Ans.} \]
2.2 Rectangular Components in Two-Dimensional Force Systems Example 12, page 1 of 3

12. Three forces support the weight \( W \) shown. Determine the value of \( F \), given that the resultant of the three forces is vertical. Also determine the value of \( W \).

Express the forces in \( x \) and \( y \) components (For clarity, the components of the unknown force, \( F \), are shown separately).

\begin{align*}
(20 \text{ N}) \sin 15^\circ & = 5.176 \text{ N} \\
(20 \text{ N}) \cos 15^\circ & = 19.32 \text{ N} \\
(120 \text{ N}) \sin 30^\circ & = 60 \text{ N} \\
(120 \text{ N}) \cos 30^\circ & = 103.92 \text{ N}
\end{align*}
2.2 Rectangular Components in Two-Dimensional Force Systems Example 12, page 2 of 3

2. Sum the components algebraically.

\[ R_x = \sum F_x : \quad R_x = 103.92 \text{ N} - 19.32 \text{ N} - F \sin 40^\circ \]  
(1)

\[ R_y = \sum F_y : \quad R_y = 60 \text{ N} + 5.176 \text{ N} + F \cos 40^\circ \]  
(2)

3. Use the fact that the resultant is known to be vertical, so \( R_x = 0 \).
   
   Eq. 1 becomes
   
   \[ R_x = 103.92 \text{ N} - 19.32 \text{ N} - F \sin 40^\circ \]

Solving gives

\[ F = 131.61 \text{ N} \quad \leftarrow \text{Ans.} \]

4. Substitute this value of \( F \) into Eq. 2 and compute \( R_y \):

\[ R_y = 60 \text{ N} + 5.176 \text{ N} + F \cos 40^\circ = 166.0 \text{ N} \]

\[ 131.61 \text{ N} \]
The resultant upward force must balance the weight $W$, so

$$W = R_y = 166.0 \text{ N}$$

$\rightarrow$ Ans.
13. The resultant, \( R \), of the forces \( A \) and \( B \) acting on the bracket is known to be a force of magnitude 300 lb making an angle of 40° with the horizontal direction as shown. Determine the magnitude of \( A \) and \( B \).

1. Express the forces in x and y components.

\[
(300 \text{ lb}) \cos 40° = 229.81 \text{ lb} \equiv R_x \text{ (x component of 300-lb resultant)}
\]

\[
(300 \text{ lb}) \sin 40° = 192.84 \text{ lb} \equiv R_y \text{ (y component of 300-lb resultant)}
\]
2.2 *Rectangular Components in Two-Dimensional Force Systems Example 13, page 2 of 2*

2) \( R_x \) is the algebraic sum of \( x \) components of \( A \) and \( B \):

\[
\begin{align*}
\updownarrow \quad R_x &= \Sigma F_x: \quad R_x = A - B \cos 70^\circ \quad (1) \\
&= 229.81 \text{ lb}
\end{align*}
\]

3) Similarly for \( R_y \):

\[
\begin{align*}
\updownarrow \quad R_y &= \Sigma F_y: \quad R_y = B \sin 70^\circ \quad (2) \\
&= 192.84 \text{ lb}
\end{align*}
\]

4) Solving Eqs. 1 and 2 simultaneously gives:

\[
\begin{align*}
A &= 300 \text{ lb} & \quad \leftarrow \text{Ans.} \\
B &= 205 \text{ lb} & \quad \leftarrow \text{Ans.}
\end{align*}
\]
14. To support the 100-N block as shown, the resultant of the 50-N force and the force $F$ must be a 100-N force directed horizontally to the right. Determine $F$ and $\theta$.

Express the forces in x and y components.

$$(50 \text{ N}) \cos 60^\circ = 25 \text{ N}$$

$$(50 \text{ N}) \sin 60^\circ = 43.30 \text{ N}$$
2.2 Rectangular Components in Two-Dimensional Force Systems Example 14, page 2 of 2

2) Sum the components algebraically.

\[ R_x = \sum F_x : R_x = F \cos \theta + 25 \text{ N} \]  

\[ R_y = \sum F_y : R_y = F \sin \theta - 43.30 \text{ N} \]  

Because the resultant is known to be horizontal, \( R_y = 0 \), and the magnitude \( R \) is thus equal to the horizontal component \( R_x \) alone, that is, \( R = R_x \). We also know, however, that the magnitude of the resultant is 100 N, so \( R = R_x = 100 \text{ N} \). Thus Eqs. 1 and 2 become

\[ 100 \text{ N} = F \cos \theta + 25 \text{ N} \]  

\[ 0 = F \sin \theta - 43.30 \text{ N} \]  

The best way to solve these equations is to use a calculator that can solve two simultaneous nonlinear equations. Alternatively, solve Eq. 4 for \( F \):

\[ F = \frac{43.30 \text{ N}}{\sin \theta} \]  

And then substitute for \( F \) in Eq. 3:

\[ 100 \text{ N} = F \cos \theta + 25 \text{ N} \]

Substituting

\[ \frac{\cos \theta}{\sin \theta} = \frac{1}{\tan \theta} \]

gives

\[ 100 \text{ N} = \frac{43.30 \text{ N}}{\tan \theta} + 25 \text{ N} \]

and solving gives

\[ \theta = 30.0^\circ \]

Using the result in Eq. 5 gives

\[ F = \frac{43.30 \text{ N}}{\sin \theta} = 86.6 \text{ N} \]

\[ \begin{array}{c}
86.6 \text{ N} \\
\downarrow 30^\circ \\
\end{array} \]

\[ \text{Ans.} \]