4.2 Moments in Three-Dimensional Force Systems
1. Use the cross-product definition of the moment of a force to determine the moment of the force about point A. Also, compare the sign of the result with that obtained from the scalar definition of positive moment, \( M = Fd \).

2. Express the force in rectangular components.

1. Introduce a position vector \( \mathbf{r}_{AB} \) with tail at A and head at B.

\[
\mathbf{r}_{AB} = \{-200\mathbf{i}\} \text{ mm}
\]

2. \( F = 50 \text{ N} \)

\[
\begin{align*}
(50 \text{ N}) \cos 30° &= 43.30 \text{ N} \\
(50 \text{ N}) \sin 30° &= 25 \text{ N}
\end{align*}
\]

\[
\mathbf{F} = \{43.30\mathbf{i} - 25\mathbf{j}\} \text{ N} \quad (1)
\]
Use the vector cross product to compute the moment about point A.

\[ M_A = \mathbf{r}_{AB} \times \mathbf{F} \]

\[ = \{-200\mathbf{i}\} \text{ mm} \times \{43.30\mathbf{i} - 25\mathbf{j}\} \text{ N} \]

\[ = \{-200(43.30) \mathbf{i} \mathbf{i} + (-200)(-25) \mathbf{i} \mathbf{j} \} \text{ N-mm} \]

\[ = 0, \text{ because cross product of parallel vectors is zero} \]

Apply the right-hand rule.

\[ i \times j = +k \]

\[ i \times k = -j \]

Assign a plus sign if the product is in the order indicated by the arrowheads; minus sign otherwise.

\[ M_A = (-200 \text{ mm})(-25 \text{ N})k \]

\[ = \{5000k\} \text{ N-mm} \]

\[ = \{5k\} \text{ N-m} \quad \leftarrow \text{Ans.} \]
4.2 Moments in Three-Dimensional Force Systems Example 1, page 3 of 3

7) Display the $\mathbf{M}_A$ vector (a double-headed arrow)

$\mathbf{M}_A = \begin{bmatrix} +5 \mathbf{k} \end{bmatrix} \text{ N} \cdot \text{m}$

8) The 50-N force tends to rotate the wrench counterclockwise about the axis (through A) defined by the unit vector $\mathbf{k}$. By the right-hand rule, this definition of positive moment reduces to the usual sign convention for positive moment in coplanar problems.
4.2 Moments in Three-Dimensional Force Systems Example 2, page 1 of 4

2. A force $F = 20$ N is applied to the end of a string of length $L$. The other end of the string is tied to the handle of a wrench as shown. Use the cross-product definition of the moment to determine the moment of $F$ about point A. Discuss the effect of distance $L$ on your answer.

Introduce a position vector $\mathbf{r}_{AC}$ with head at C and tail at A.

$\mathbf{F} = 20$ N

$\mathbf{200}$ mm

$30^\circ$
4.2 Moments in Three-Dimensional Force Systems Example 2, page 2 of 4

2. Determine the rectangular components of $\mathbf{r}_{AC}$.

3. $\mathbf{r}_{AC} = (L \cos 30° + 200 \text{ mm}) \mathbf{i} - (L \sin 30°) \mathbf{j}$  \hspace{1cm} (1)

4. $L \sin 30°$

5. $L \cos 30°$

6. Determine the rectangular components of the force $\mathbf{F}$

7. $\mathbf{F} = -(20 \text{ N}) \cos 30° \mathbf{i} - (20 \text{ N}) \sin 30° \mathbf{j}$  \hspace{1cm} (2)
4.2 Moments in Three-Dimensional Force Systems Example 2, page 3 of 4

7 Use the cross product to compute the moment.

\[ \mathbf{M}_A = \mathbf{r}_{AC} \times \mathbf{F} \]

\[ = \{(-L \cos 30° - 200)i - L \sin 30°j\} \text{ mm} \times \{-20 \cos 30°i - 20 \sin 30°j\} \text{ N} \]

\[ = [(-L \cos 30° - 200)(-20 \cos 30°)(\mathbf{i} \times \mathbf{i}) \]

\[ = 0 \]

\[ + (-L \cos 30° - 200)(-20 \sin 30°)(\mathbf{i} \times \mathbf{j}) \]

\[ = \mathbf{k} \]

\[ + (-L \sin 30°)(-20 \cos 30°)(\mathbf{j} \times \mathbf{i}) \]

\[ = -\mathbf{k} \]

\[ + (-L \sin 30°)(-20 \sin 30°)(\mathbf{j} \times \mathbf{j}) \text{ N·mm} \]

\[ = 0 \]

\[ = \mathbf{k}[(-L \cos 30° - 200)(-20 \sin 30°) - (-L \sin 30°)(-20 \cos 30°)] \text{ N·mm} \]

\[ = \mathbf{k}[(-L \cos 30°)(-20 \sin 30°) - 200(-20 \sin 30°)

- (-L \sin 30°)(-20 \cos 30°)] \text{ N·mm} \]

The terms involving \( L \) drop out

\[ = \{2000\mathbf{k}\} \text{ N·mm} = \{2\mathbf{k}\} \text{ N·m} \]

\( \leftarrow \text{Ans.} \)
Discussion: Distance \( L \) does not appear in the answer for \( \mathbf{M}_A \). Thus the result is valid for all values of \( L \), and so the head of the position vector can be located anywhere (for example, points 1, 2, 3, or C in the figure) on the line of action of the force.

Note: All position vectors and force vectors are in the xy plane.
3. A shower/bathtub grab bar is being pulled by a force $F = 30$ lb as shown. Determine the moment of $F$ about the support $A$. Also determine the coordinate direction angles of the moment vector and interpret the result.
Introduce a position vector with tail at A and head at B.

\[ \mathbf{r}_{AB} = \{-16i + 8j + 5k\} \text{ in.} \quad (1) \]
4.2 Moments in Three-Dimensional Force Systems Example 3, page 3 of 6

2 Determine the rectangular components of the force \( F \).

6 \( F_x = -(30 \text{ lb})(\sin 60^\circ)(\cos 40^\circ) \)
\[ = -19.90 \text{ lb} \quad (4) \]

5 \( F_z = (30 \text{ lb})(\sin 60^\circ)(\sin 40^\circ) \)
\[ = 16.70 \text{ lb} \quad (3) \]

4 \((30 \text{ lb}) \sin 60^\circ\)

3 \( F_y = -(30 \text{ lb}) \cos 60^\circ \)
\[ = -15 \text{ lb} \quad (2) \]

Component form of \( F \), from Eqs. 2-4:
\[ \mathbf{F} = \{-19.90\mathbf{i} - 15\mathbf{j} + 16.70\mathbf{k}\} \text{ lb} \quad (5) \]
Calculate the moment.

\[ M_A = \mathbf{r}_{AB} \times \mathbf{F} \]

\[ = \{-16\mathbf{i} + 8\mathbf{j} + 5\mathbf{k}\} \times \{-19.90\mathbf{i} - 15\mathbf{j} + 16.70\mathbf{k}\} \]

Because both vectors each have three non-zero components, evaluating the cross products is easier if we use the determinant form.

\[
M_A = \begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
-16 & 8 & 5 \\
-19.90 & -15 & 16.70
\end{vmatrix}
\]

Expand the determinant in terms of the first row, remembering to insert the minus sign for the \(\mathbf{j}\) term.

\[
M_A = \begin{vmatrix}
\mathbf{i} & 8 & 5 \\
-15 & 16.70 & -16 \\
-19.90 & 16.70 & 5
\end{vmatrix}
+ \begin{vmatrix}
\mathbf{i} & 8 & 5 \\
-15 & 16.70 & -16 \\
-19.90 & 16.70 & 8
\end{vmatrix}
\]

\[
= \mathbf{i}[8(16.70) - 5(-15)] - \mathbf{j}[-16(16.70) - 5(-19.90)] + \mathbf{k}[-16(-15) - 8(-19.90)]
\]

\[
= \{208.6\mathbf{i} + 167.7\mathbf{j} + 399.2\mathbf{k}\} \text{ lb·in.}
\]
Observation: The above procedure is tedious and error prone. A much better approach is to use a calculator with a built-in function for evaluating the cross product of two vectors. If such a calculator is available, all you need to do is enter the components of the vectors and then let the calculator perform the arithmetic. As a result, typical errors such as missing a minus sign or mis-copying a number from one line to the next are avoided.

To determine the coordinate direction angles, first determine the magnitude of the moment.

\[ M_A = \sqrt{(208.6)^2 + (167.7)^2 + (399.2)^2} \]

\[ = 480.6 \text{ lb-in.} \]

Coordinate direction angles

\[ \alpha = \cos^{-1} \frac{M_{Ax}}{M_A} = \cos^{-1} \frac{208.6}{480.6} = 64.3^\circ \]

\[ \beta = \cos^{-1} \frac{M_{Ay}}{M_A} = \cos^{-1} \frac{167.7}{480.6} = 69.6^\circ \]

\[ \gamma = \cos^{-1} \frac{M_{Az}}{M_A} = \cos^{-1} \frac{399.2}{480.6} = 33.8^\circ \]
4.2 Moments in Three-Dimensional Force Systems Example 3, page 6 of 6

Interpretation: The force $F = 30$ lb produces a moment of $480.6$ lb·in. about point A. This moment tends to rotate the grab bar about an axis defined by the moment vector.

Axis of rotation

$M_A = 480.6$ lb·in.

Ans.

$F = 30$ lb
4.2 Moments in Three-Dimensional Force Systems Example 4, page 1 of 3

4. A force \( F = 15 \text{ N} \) acting parallel to the \( z \) axis is applied to the handle of a socket wrench to turn a bolt at A. Determine the moment of the force about the point A. Also, state which component of the moment tends to turn the bolt.
4.2 Moments in Three-Dimensional Force Systems Example 4, page 2 of 3

1. Introduce a position vector with head at B and tail at A.
   \[ \mathbf{r}_{AB} = \{80i - 100j\} \text{ mm} \]

2. Calculate the moment.
   \[ \mathbf{M}_A = \mathbf{r}_{AB} \times \mathbf{F} \]
   \[ = (80i - 100j) \times \{-15k\} \] (F points in negative z-direction.)
   \[ = 80(-15)(i \times k) + (-100)(-15)(j \times k) \]
   \[ = 80(-15)i - 80(-15)j + (-100)(-15)i + (-100)(-15)j \]
   \[ = -1200(\mathbf{j}) + 1500(\mathbf{i}) \]
   \[ = \{1500\mathbf{i} + 1200\mathbf{j}\} \text{ N-mm} \]
   \[ = \{1.5\mathbf{i} + 1.2\mathbf{j}\} \text{ N-m} \] ← Ans.
4.2 Moments in Three-Dimensional Force Systems Example 4, page 3 of 3

Display the moment vector.

The component that tends to rotate the shaft AC of the wrench about the x axis (and thus turn the bolt) is

\[ M_{Ax} = 1.5 \text{ N} \cdot \text{m} \]
Answer.

The component that tends to rotate the shaft AC of the wrench about the x axis (and thus turn the bolt) is

\[ M_{Ax} = 1.5 \text{ N} \cdot \text{m} \quad \leftarrow \text{Ans.} \]
5. Pulley B is used to drive pulley C. Determine the resultant moment about bearing A produced by the belt forces acting on pulley B. Also, interpret your result.

Belt forces

Q = 55 N
P = 30 N
Radius = 70 mm
40 mm
4.2 Moments in Three-Dimensional Force Systems Example 5, page 2 of 6

1 Introduce position vectors with tails at A and heads at E and F.

2 From the figure,
   \[ \mathbf{r}_{AE} = \{40i, -70j\} \text{ mm} \quad (1) \]
In component form, from Eqs. 2 and 3,
\[ \mathbf{r}_{AF} = \{40i + 60.62j - 35k\} \text{ mm} \] (4)
Express the forces in rectangular components.

\[
Q = (55 \text{ N}) \sin 30^\circ \hat{j} + (55 \text{ N}) \cos 30^\circ \hat{k}
\]

\[
= \{27.5 \hat{j} + 47.63 \hat{k}\} \text{ N}
\]  \hspace{1cm} (5)

\[
P = \{30 \hat{k}\} \text{ N}
\]  \hspace{1cm} (6)
4.2 Moments in Three-Dimensional Force Systems Example 5, page 5 of 6

Calculate the resultant moment.

\[ \mathbf{M}_A = \mathbf{r}_{AF} \times \mathbf{Q} + \mathbf{r}_{AE} \times \mathbf{P} \]

\[ \mathbf{M}_A = \begin{vmatrix} i & j & k \\ 40 & 60.62 & -35 \\ 0 & 27.5 & 47.63 \end{vmatrix} + \begin{vmatrix} i & j & k \\ 40 & -70 & 0 \\ 0 & 0 & 30 \end{vmatrix} \]

\[ = i \begin{vmatrix} 60.62 & -35 \\ 27.5 & 47.63 \end{vmatrix} - j \begin{vmatrix} 40 & -35 \\ 0 & 47.63 \end{vmatrix} + k \begin{vmatrix} 40 & 60.62 \\ 0 & 27.5 \end{vmatrix} \]

\[ + i \begin{vmatrix} -70 & 0 \\ 0 & 30 \end{vmatrix} - j \begin{vmatrix} 40 & 0 \\ 0 & 30 \end{vmatrix} + k \begin{vmatrix} 40 & -70 \\ 0 & 0 \end{vmatrix} \]

\[ = i[60.62(47.63) - (-35)(27.5)] - j[40(47.63) - (-35)(0)] + k[40(27.5) - 60.62(0)] \]

\[ + i[(-70)(30) - 0(0)] - j[40(30) - 0(0)] + k[40(0) - (-70)(0)] \]

\[ = \begin{vmatrix} 1750 & -3105 & 1100 \end{vmatrix} \text{ N-mm} \]

\[ = \begin{vmatrix} 1.750 & -3.105 & 1.100 \end{vmatrix} \text{ N-m} \quad \leftarrow \text{Ans.} \]
The magnitude of the moment is
\[ M_A = \sqrt{(1.750)^2 + (-3.105)^2 + (1.100)^2} \]
\[ = 3.73 \text{ N} \cdot \text{m} \]

Interpretation: The belt forces acting on pulley B tend to rotate the entire structure with a 3.73 N-m moment about an axis defined by the direction of the moment vector \( M_A \). The 1.750 N-m \( x \) component of \( M_A \) is the component of moment that rotates the shaft and drives pulley C.
6. A child on a bicycle collides with a mailbox and exerts the force $F$ shown. If the base of the pole at $O$ will fail if the magnitude of the moment there exceeds 60 N·m, determine if the mailbox will fall over.

$F = \{80i + 12j - 10k\}$ N

\[250 \text{ mm}\]
\[75 \text{ mm}\]

A

O

\[900 \text{ mm}\]

z

250 mm

75 mm

x
4.2 Moments in Three-Dimensional Force Systems Example 6, page 2 of 3

1. Introduce a position vector with tail at O and head at A.

\[ \mathbf{r}_{OA} = \{250\mathbf{i} + 900\mathbf{j} + 75\mathbf{k}\} \text{ mm} \]

2. Calculate the moment.

\[ \mathbf{M}_O = \mathbf{r}_{OA} \times \mathbf{F} \]

\[
\begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
250 & 900 & 75 \\
80 & 12 & -10 \\
\end{vmatrix}
\]

\[
= \mathbf{i} [900(-10) - (12)(75)] - \mathbf{j} [250(-10) - 80(75)] + \mathbf{k} [250(12) - 80(900)]
\]

\[
= \{-9900\mathbf{i} + 8500\mathbf{j} - 69000\mathbf{k}\} \text{ N-mm}
\]

\[
= \{-9.9\mathbf{i} + 8.5\mathbf{j} - 69.0\mathbf{k}\} \text{ N-m}
\]
4.2 Moments in Three-Dimensional Force Systems Example 6, page 3 of 3

3. Magnitude of moment

\[ M_0 = \sqrt{(-9.9)^2 + (8.5)^2 + (-69.0)^2} \]

\[ = 70.2 \text{ N} \cdot \text{m} \]

4. Because 70.2 N·m exceeds the 60 N·m maximum allowable moment, the mailbox will fall over. ←Ans.
7. Copper tubing emerges from the wall at A and is subjected to a force F at its free end B. The tubing will fail if the magnitude of the moment at A exceeds 3 N·m. Determine the largest value of the force F that can be applied to the free end of the tubing.
4.2 Moments in Three-Dimensional Force Systems Example 7, page 2 of 6

1. Introduce a position vector $\mathbf{r}_{AB}$ with tail at A and head at B.
Determine the components of $r_{AB}$.

3. $r_{ABx} = 200 \text{ mm} + 250 \text{ mm} = 450 \text{ mm}$
4.2 Moments in Three-Dimensional Force Systems Example 7, page 4 of 6

To determine the y and z components of \( \mathbf{r}_{AB} \), consider a view from the positive x axis.

\[
\mathbf{r}_{AB} = \{450 \mathbf{i} - 172.1 \mathbf{j} + 245.7 \mathbf{k}\} \text{ mm} \quad (4)
\]

\[
\mathbf{r}_{ABz} = (300 \text{ mm}) \cos 35^\circ = 245.7 \text{ mm} \quad (3)
\]

\[
\mathbf{r}_{ABy} = -(300 \text{ mm}) \sin 35^\circ = -172.1 \text{ mm} \quad (2)
\]
4.2 Moments in Three-Dimensional Force Systems Example 7, page 5 of 6

8 Determine the components of $F$.

\[ F_x = F \sin 40^\circ \cos 30^\circ = 0.5567F \quad (7) \]
\[ F_y = -F \cos 40^\circ = -0.7660F \quad (5) \]
\[ F_z = F \sin 40^\circ \sin 30^\circ = 0.3214F \quad (6) \]

13 In component form, from Eqs. 5-7,

\[ F = F \{0.5567i - 0.7660j + 0.3214k\} \quad (8) \]
4.2 Moments in Three-Dimensional Force Systems Example 7, page 6 of 6

14 Calculate the resultant moment.

\[ M_A = \mathbf{r}_{AB} \times \mathbf{F} \]

\[
\begin{vmatrix}
  \mathbf{i} & \mathbf{j} & \mathbf{k} \\
 450 & -172.1 & 245.7 \\
 0.5567\mathbf{F} & -0.7660\mathbf{F} & 0.3214\mathbf{F} \\
\end{vmatrix}
\]

\[
= \mathbf{i} \begin{vmatrix} -172.1 & 245.7 \end{vmatrix} - \mathbf{j} \begin{vmatrix} 450 & 245.7 \end{vmatrix} + \mathbf{k} \begin{vmatrix} 450 & -172.1 \end{vmatrix}
\]

\[
= (132.9\mathbf{F})\mathbf{i} - (7.8\mathbf{F})\mathbf{j} - (248.9\mathbf{F})\mathbf{k}
\]

15 Compute the magnitude.

\[
M_A = \sqrt{(132.9\mathbf{F})^2 + (-7.8\mathbf{F})^2 + (-248.9\mathbf{F})^2}
\]

\[
= 282.3\mathbf{F} \text{ N}\cdot\text{mm}
\]

\[
= 0.2823\mathbf{F} \text{ N}\cdot\text{m}
\]

16 Equate \( M_A \) to the largest allowable moment, 3 N\cdot\text{m}:

\[
M_A = 3 \text{ N}\cdot\text{m}
\]

\[
0.2823\mathbf{F} \text{ N}\cdot\text{m}
\]

Solving gives

\[ F = 10.6 \text{ N} \quad \text{Ans.} \]
8. Two forces, $P = 60 \text{ N}$ and $Q = 80 \text{ N}$ act on the vertices of a cube as shown. Determine the moment of each force about point $O$, if the length of each edge of the cube is $2 \text{ m}$. Also, determine the shortest distance from $O$ to the line $BF$. 

Each edge is $2 \text{ m}$ long.
To determine the moment of the force P about point O, we have several choices of position vector (Recall that the head of the vector can lie anywhere on the line of action of P).

1. Select the position vector with the simpler form.

\[ \mathbf{r}_{OD} = \{2i + 2j + 2k\} \text{ m} \]

\[ \mathbf{r}_{OE} = \{2j\} \text{ m} \]  

(1) Simpler (only one component)
The force \( P \) has magnitude 60 N and points in the direction from D to E, so

\[
P = (60 \, \text{N}) \frac{\mathbf{r}_{DE}}{r_{DE}}
\]

\[
= (60 \, \text{N}) \frac{-2\mathbf{i} - 2\mathbf{k}}{\sqrt{(-2)^2 + (-2)^2}}
\]

\[
= \{-42.43\mathbf{i} - 42.43\mathbf{k}\} \, \text{N}
\]
5. Calculate the moment.

\[ \mathbf{M}_{OP} = \mathbf{r}_{OE} \times \mathbf{P} \]

\[ = \{2\mathbf{j}\} \times \{-42.43\mathbf{i} - 42.43\mathbf{k}\} \]

\[ = (2)(-42.43)(\mathbf{j} \times \mathbf{i}) - (2)(42.43)(\mathbf{j} \times \mathbf{k}) \]

\[ = -\mathbf{k} = \mathbf{i} \]

\[ = \{-84.86\mathbf{i} + 84.86\mathbf{k}\} \text{ N\cdotm} \]

Since \( \mathbf{r}_{OE} \) has only one component, it is easier to calculate cross products of base vectors individually rather than use the determinant approach to calculating the cross product.

\[ \text{Ans.} \]
4.2 Moments in Three-Dimensional Force Systems Example 8, page 5 of 6

To determine the moment of the force $Q$ about point $O$, we could introduce either position vector $\mathbf{r}_{OB}$ or $\mathbf{r}_{OF}$. Let's arbitrarily choose $\mathbf{r}_{OB}$.

$$\mathbf{r}_{OB} = (2\mathbf{i} + 2\mathbf{k}) \text{ m}$$

Calculate the moment.

$$M_{OQ} = \mathbf{r}_{OB} \times \mathbf{Q}$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 0 & 2 \\ 0 & -56.57 & 56.57 \end{vmatrix}$$

$$= \mathbf{i} \begin{vmatrix} 0 & 2 \\ -56.57 & 56.57 \end{vmatrix} - \mathbf{j} \begin{vmatrix} 2 & 0 \\ 0 & -56.57 \end{vmatrix} + \mathbf{k} \begin{vmatrix} 2 & 0 \\ -56.57 & 56.57 \end{vmatrix}$$

$$= [113.14\mathbf{i} - 113.14\mathbf{j} - 113.14\mathbf{k}] \text{ N}\cdot\text{m} \quad (3) \quad \leftarrow \text{Ans.}$$
Finally, to determine the shortest distance from point O to line FB, consider the plane defined by O and FB:

From the figure it is clear that of the various distances $d_1$, $d_2$, $d_3$, ..., from O to line FB, the perpendicular distance $d$ is the shortest. But we also know that, by the scalar definition of moment, the moment of Q about point O equals the perpendicular distance from O to the line of action of Q times Q. That is,

$$M_{QO} = Qd$$

Thus,

$$d = \frac{M_{QO}}{Q}$$

$$= \frac{\sqrt{(113.14)^2 + (-113.14)^2 + (-113.14)^2}}{80}$$

$$= 2.45 \text{ m} \quad \leftarrow \text{Ans.}$$
4.2 Moments in Three-Dimensional Force Systems Example 9, page 1 of 6

9. Determine the moment about the screw at A of the force \( F = 2 \) N applied to the sheet-metal bracket shown. Also, determine the shortest distance from A to the line connecting B and C.
To calculate the moment we could use either the position vector $\mathbf{r}_{AB}$ or $\mathbf{r}_{AC}$.

1. $\mathbf{r}_{AC}$ has a single component,

\[
\mathbf{r}_{AC} = -(50 \text{ mm} + 60 \text{ mm}) \mathbf{i} = \{-110\mathbf{i}\} \text{ mm}
\]  

while $\mathbf{r}_{AB}$ has two components, so let's choose the simpler, $\mathbf{r}_{AC}$.
4.2 Moments in Three-Dimensional Force Systems Example 9, page 3 of 6

Next, we need to calculate the components of the force, \( F \). To do this, we first need to calculate the coordinates of point \( B \). By inspection, the \( x \) coordinate of \( B \) is 60 mm.

To find the \( y \) and \( z \) coordinates of point \( B \), consider a view from the positive \( x \)-axis.

1. \( y \) coordinate of \( B \)
   \[ y = 30 \text{ mm} \]
   \[ y = 70 \text{ mm} \]
   \[ y = 142.26 \text{ mm} \]

2. \( z \) coordinate of \( B \)
   \[ z = 90.63 \text{ mm} + 80 \text{ mm} \]
   \[ z = 170.63 \text{ mm} \]

3. \( (100 \text{ mm}) \sin 25^\circ = 42.26 \text{ mm} \)

4. \( (100 \text{ mm}) \cos 25^\circ = 90.63 \text{ mm} \)

5. \( y \) coordinate of \( B \)
   \[ y = -30 \text{ mm} - 70 \text{ mm} - 42.26 \text{ mm} \]
   \[ y = -142.26 \text{ mm} \]
To determine the components of the force, introduce the position vector $\mathbf{r}_{BC}$.

$$\mathbf{r}_{BC} = (-50 \text{ mm} - 60 \text{ mm})\hat{i} + [0 - (-142.26 \text{ mm})]\hat{j} + (0 - 170.63 \text{ mm})\hat{k}$$

$$= \{-110\hat{i} + 142.26\hat{j} - 170.63\hat{k}\} \text{ mm} \quad (2)$$

The force $\mathbf{F}$ has magnitude 2 N and points from B to C so

$$\mathbf{F} = 2 \text{ N} \frac{\mathbf{r}_{BC}}{\mathbf{r}_{BC}} = (2 \text{ N}) \frac{-110\hat{i} + 142.26\hat{j} - 170.63\hat{k}}{\sqrt{(-110)^2 + (142.26)^2 + (-170.63)^2}}$$

$$= \{-0.887\hat{i} + 1.148\hat{j} - 1.377\hat{k}\} \text{ N}$$
4.2 Moments in Three-Dimensional Force Systems Example 9, page 5 of 6

Calculate the moment.

\[ \mathbf{M}_A = \mathbf{r}_{AC} \times \mathbf{F} \]

\[ = \{-110\mathbf{i}\} \times \{-0.887\mathbf{i} + 1.148\mathbf{j} - 1.377\mathbf{k}\} \]

\[ = (-110)(-0.887)(\mathbf{i} \times \mathbf{i}) + (-110)(1.148)(\mathbf{i} \times \mathbf{j}) + (-110)(-1.377)(\mathbf{i} \times \mathbf{k}) \]

\[ = 0 \quad = \mathbf{k} \quad = -\mathbf{j} \]

\[ = \{-151.4\mathbf{j} - 126.3\mathbf{k}\} \text{ N-mm} \quad (3) \quad \leftarrow \text{Ans.} \]
To determine the shortest distance between point A and line BC, consider the plane formed by A and BC.

**Magnitude of moment about A = Fd**

or,

\[
\sqrt{(-151.4)^2 + (-126.3)^2} \text{ N mm} = (2 \text{ N})d
\]

Solving gives,

\[
d = 98.6 \text{ mm}
\]

←Ans.
10. If the tension in the cable BC is $T = 80$ lb, determine the moment about point A of the cable force acting on the frame at point B. Also, determine the shortest distance from A to the line through B and C.
4.2 Moments in Three-Dimensional Force Systems Example 10, page 2 of 5

To calculate the moment, we could use either position vector \( \mathbf{r}_{AB} \) or \( \mathbf{r}_{AC} \).

1. Since \( \mathbf{r}_{AC} \) has only one component while \( \mathbf{r}_{AB} \) has three, let's use \( \mathbf{r}_{AC} \).

\[
\mathbf{r}_{AC} = \{12j\} \text{ in.} \quad (1)
\]

2. \( \mathbf{r}_{AC} \) has length 12 in.

\[
\begin{align*}
&12 \text{ in.} \\
&18 \text{ in.} \\
&10 \text{ in.} \\
&32 \text{ in.}
\end{align*}
\]
4.2 Moments in Three-Dimensional Force Systems Example 10, page 3 of 5

3 To determine the components of the force, \( T \), introduce the position vector \( r_{BC} \).

\[
\begin{align*}
    r_{BC} &= (0 - 32 \text{ in.})\mathbf{i} + (12 \text{ in.} - 10 \text{ in.})\mathbf{j} + (0 - 18 \text{ in.})\mathbf{k} \\
    &= \{-32\mathbf{i} + 2\mathbf{j} - 18\mathbf{k}\} \text{ in.}
\end{align*}
\]

The force \( T \) has magnitude 80 lb and points from B to C so

\[
T = 80 \text{ lb}
\]

4 The force \( T \) has magnitude 80 lb and points from B to C so

\[
T = (80 \text{ lb}) \frac{r_{BC}}{r_{BC}} = (80 \text{ lb}) \frac{-32\mathbf{i} + 2\mathbf{j} - 18\mathbf{k}}{\sqrt{(-32)^2 + 2^2 + (-18)^2}}
\]

\[
= \{-69.62\mathbf{i} + 4.35\mathbf{j} - 39.16\mathbf{k}\} \text{ lb}
\]
4.2 Moments in Three-Dimensional Force Systems Example 10, page 4 of 5

Calculate the moment.

\[
M_A = \mathbf{r}_{AC} \times \mathbf{T}
\]

by Eq. 1

\[
= 12 \mathbf{j} \times (-69.62 \mathbf{i} + 4.35 \mathbf{j} - 39.16 \mathbf{k})
\]

\[
= (12)(-69.62)(\mathbf{j} \times \mathbf{i}) + (12)(4.35)(\mathbf{j} \times \mathbf{j}) + (12)(-39.16)(\mathbf{j} \times \mathbf{k})
\]

\[
= -\mathbf{k} = 0 = \mathbf{i}
\]

\[
= \{-469.92 \mathbf{i} + 835.44 \mathbf{k}\} \text{ lb-in.} \quad \leftarrow \text{Ans.} \quad (2)
\]
Finally, to determine the shortest distance between point A and line BC, consider the plane formed by A and BC.

\[ \text{Magnitude of moment about A} = Td \]
\[ \text{or, } \quad \sqrt{(-469.92)^2 + (835.44)^2} = (80 \text{ lb})d \]

Solving gives,
\[ d = 11.98 \text{ in.} \]
\(\leftarrow\) Ans.