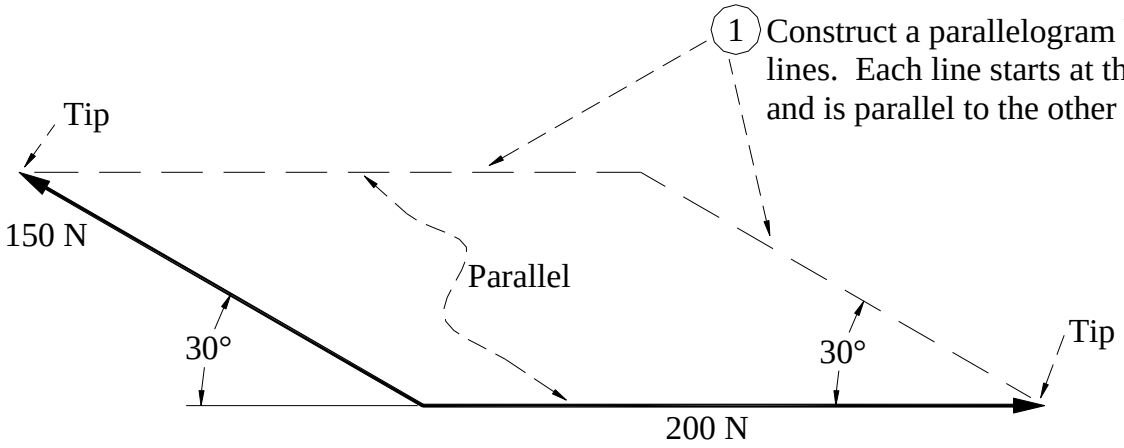
2.1 Adding Forces by the Parallelogram Law

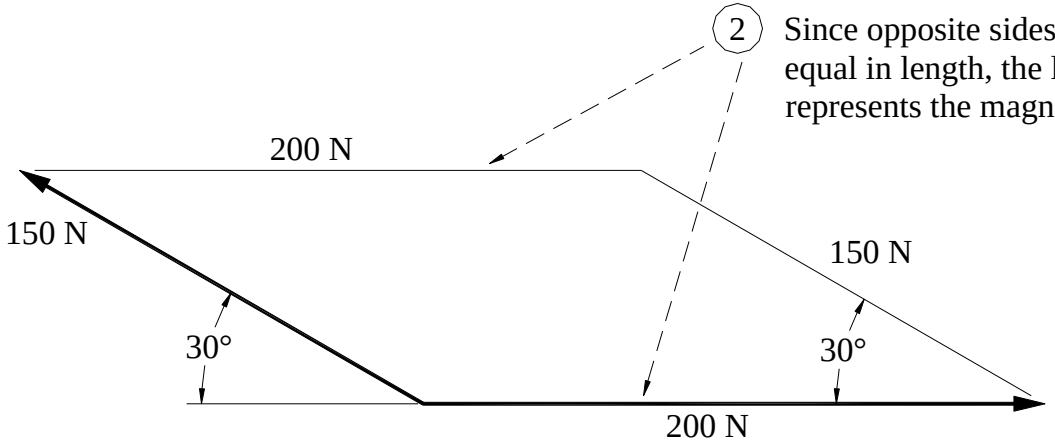
2.1 Adding Forces by the Parallelogram Law Example 1, page 1 of 4

1. Determine the magnitude and direction of the resultant of the forces shown.

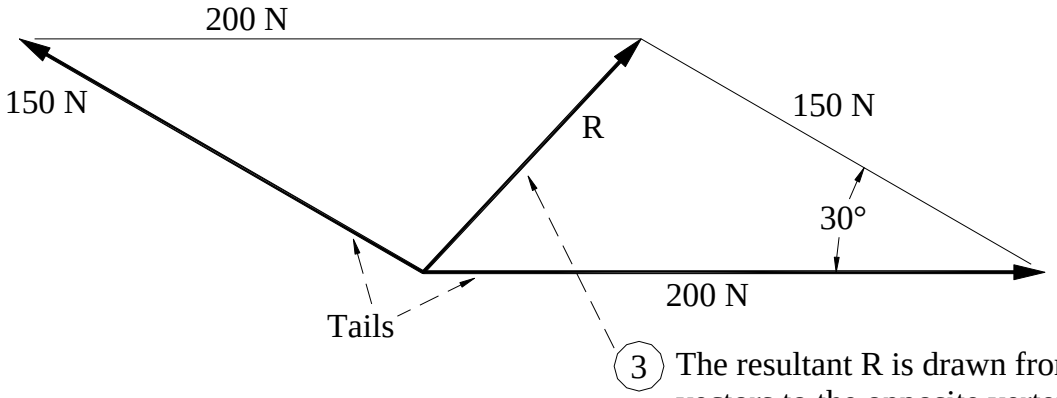


Construct a parallelogram by drawing two lines. Each line starts at the tip of one vector and is parallel to the other vector.

2.1 Adding Forces by the Parallelogram Law Example 1, page 2 of 4

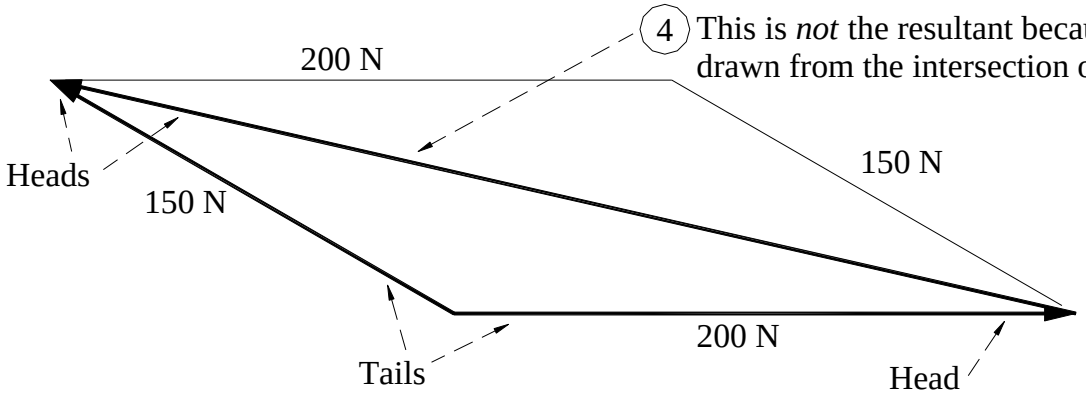


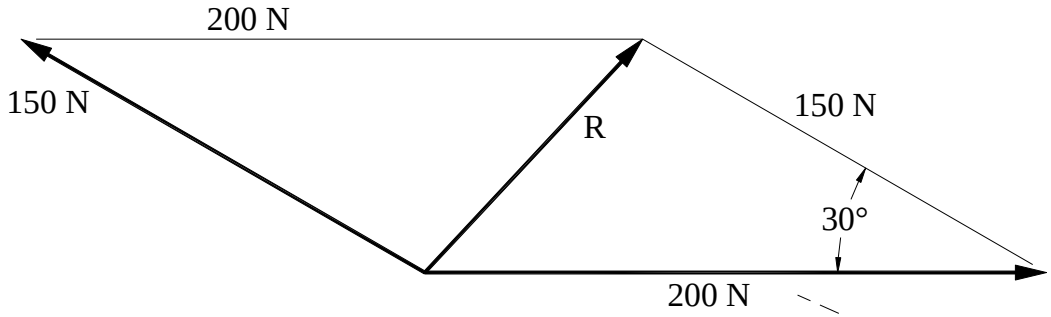
Since opposite sides of a parallelogram are equal in length, the length of each line represents the magnitude of the vector opposite.

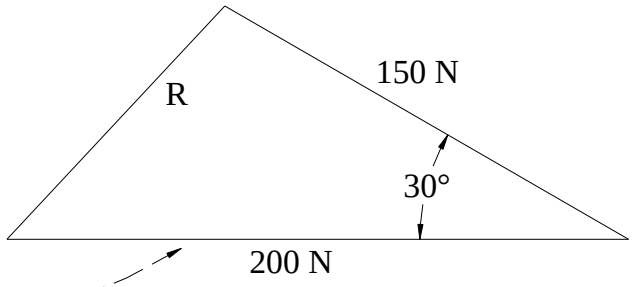


The resultant R is drawn from the *tails* of the vectors to the opposite vertex of the parallelogram.

2.1 Adding Forces by the Parallelogram Law Example 1, page 3 of 4

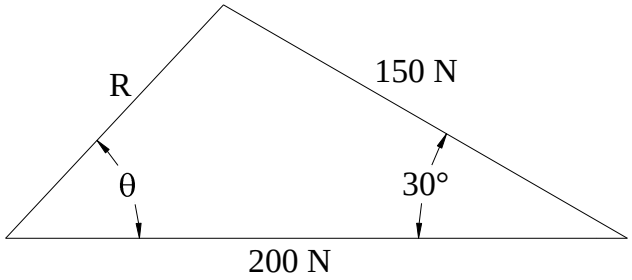




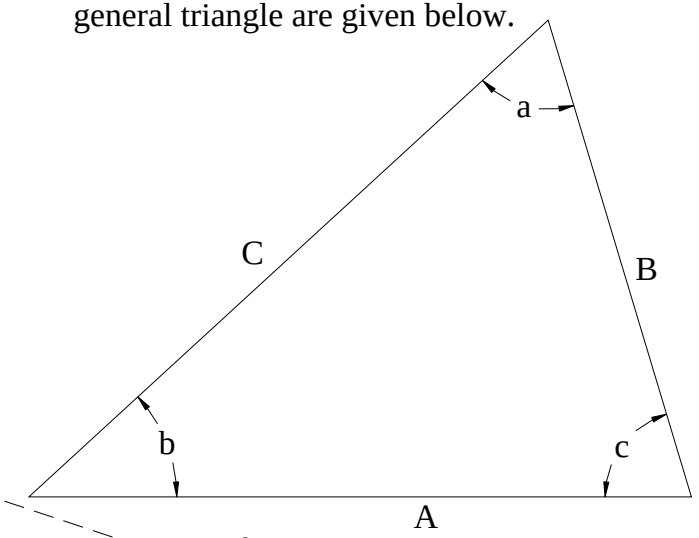


5 To calculate the magnitude and direction of R, consider the triangle formed by one half of the parallelogram.

2.1 Adding Forces by the Parallelogram Law Example 1, page 4 of 4



Trigonometric formulas for a general triangle are given below.



6 Use trigonometry to calculate the magnitude and direction of the resultant.

$$R^2 = (200\text{ N})^2 + (150\text{ N})^2 - 2(200\text{ N})(150\text{ N})\cos 30°$$

The result is

$$R = 102.66\text{ N} \quad \leftarrow \text{Ans.}$$

Law of cosines

$$C^2 = A^2 + B^2 - 2AB\cos c$$

$$\frac{\sin\theta}{150\text{ N}} = \frac{\sin 30°}{R} = 102.66\text{ N}$$

Solving gives

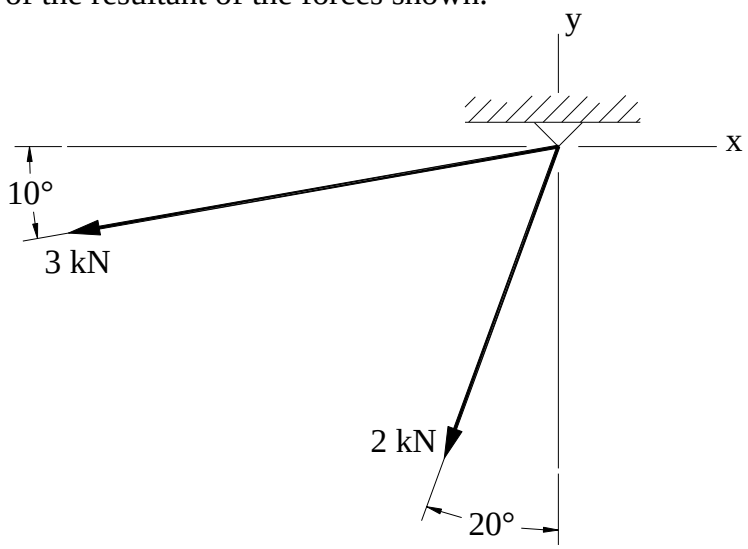
$$\theta = 46.9° \quad \leftarrow \text{Ans.}$$

Law of sines

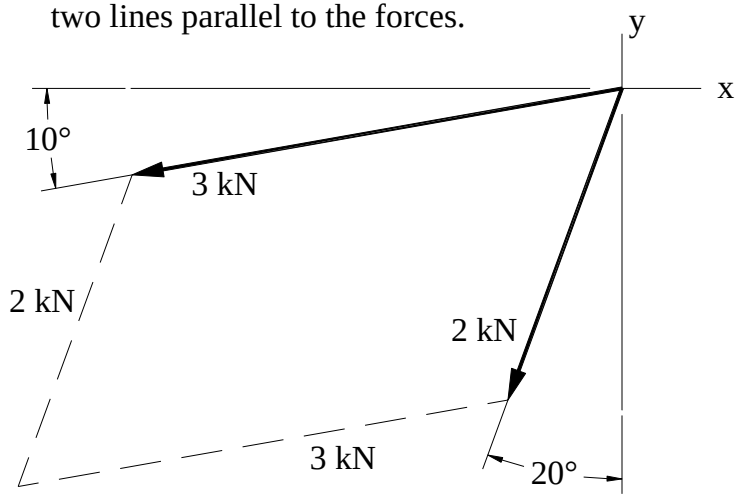
$$\frac{\sin a}{A} = \frac{\sin b}{B} = \frac{\sin c}{C}$$

2.1 Adding Forces by the Parallelogram Law Example 2, page 1 of 3

2. Determine the magnitude and direction of the resultant of the forces shown.

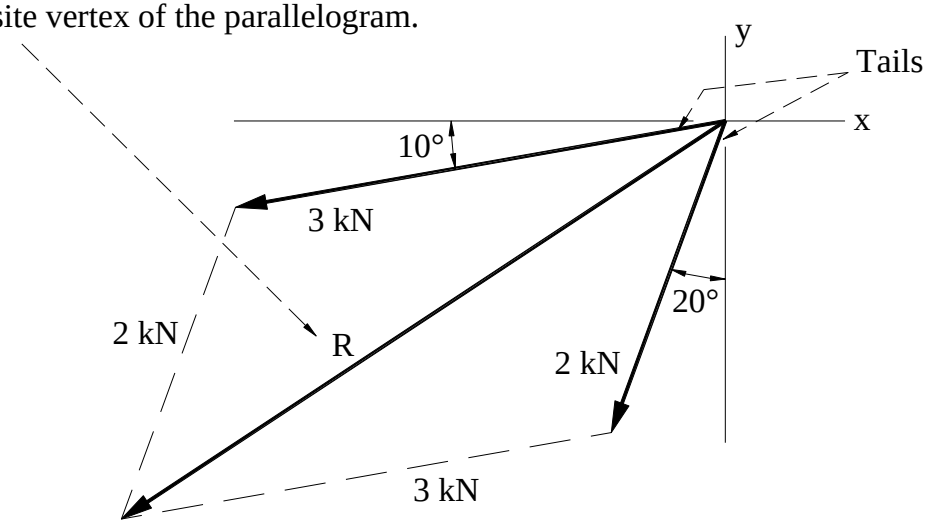


(1) Construct a parallelogram by drawing two lines parallel to the forces.

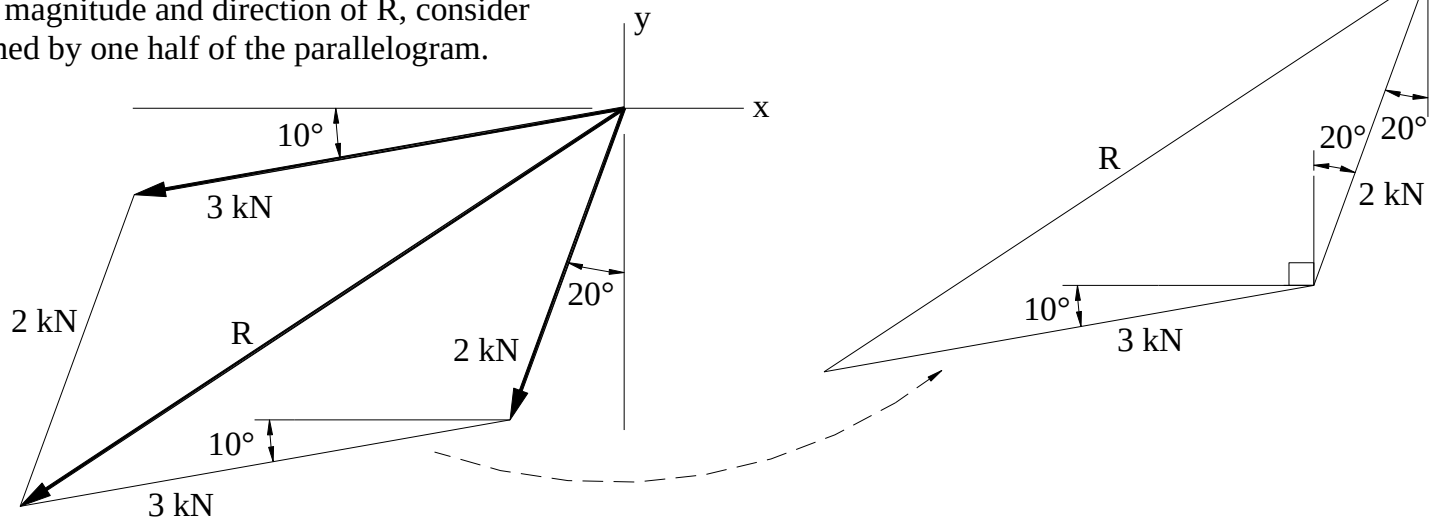


2.1 Adding Forces by the Parallelogram Law Example 2, page 2 of 3

(2) Draw the resultant R from the tails of the vectors to the opposite vertex of the parallelogram.

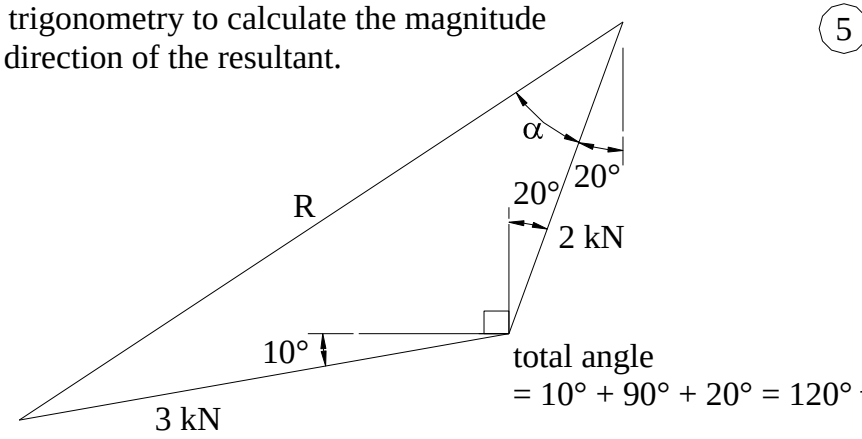


(3) To calculate the magnitude and direction of R, consider the triangle formed by one half of the parallelogram.



2.1 Adding Forces by the Parallelogram Law Example 2, page 3 of 3

④ Use trigonometry to calculate the magnitude and direction of the resultant.



total angle
= 10° + 90° + 20° = 120°

⑤ Law of cosines

$$R^2 = (3\text{ kN})^2 + (2\text{ kN})^2 - 2(3\text{ kN})(2\text{ kN})\cos 120°$$

$R = 4.359$ kN ←Ans.

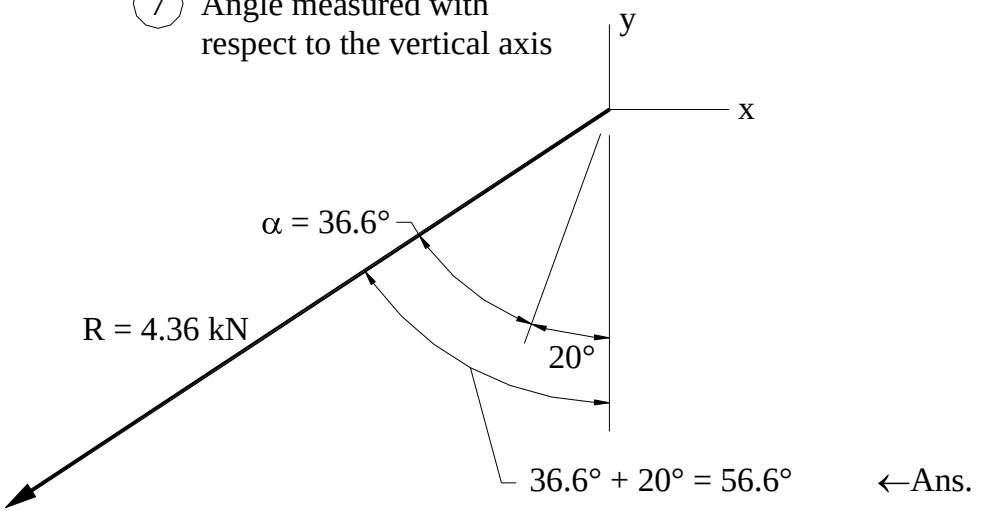
⑥ Law of sines

$$\frac{\sin\alpha}{3\text{ kN}} = \frac{\sin 120°}{R} = 4.359\text{ kN}$$

Solving gives

$\alpha = 36.6°$

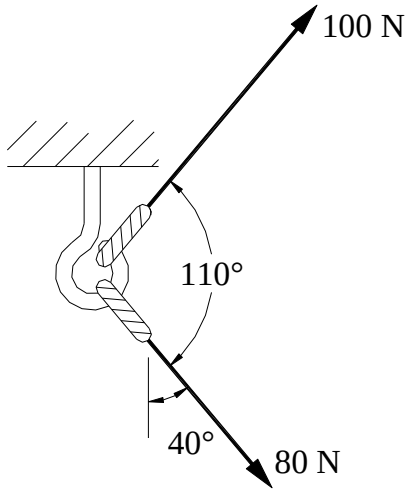
⑦ Angle measured with respect to the vertical axis



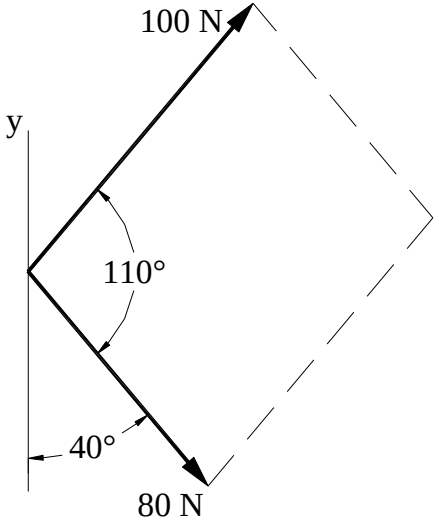
36.6° + 20° = 56.6° ←Ans.

2.1 Adding Forces by the Parallelogram Law Example 3, page 1 of 3

3. Determine the magnitude and direction of the resultant force.

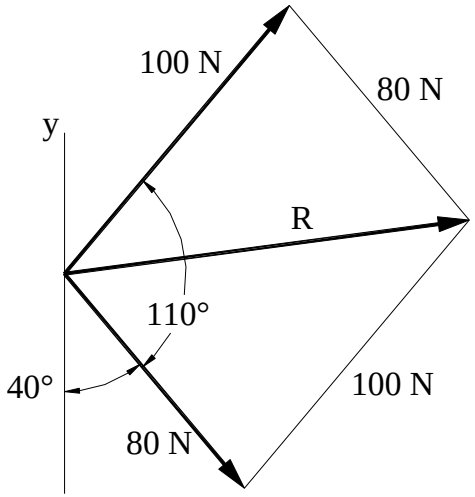


(1) Construct a parallelogram

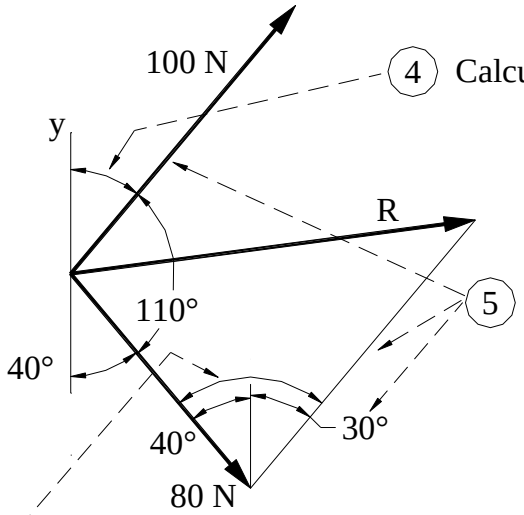


2.1 Adding Forces by the Parallelogram Law Example 3, page 2 of 3

(2) Draw the resultant R from the tails of the vectors to the opposite vertex of the parallelogram.



(3) To calculate R, consider the triangle formed by the lower half of the parallelogram.



(4) Calculate angle

$180° - 110° - 40° = 30°$

(5) Parallel lines make 30° angle with vertical direction

(6) Calculate angle

$30° + 40° = 70°$

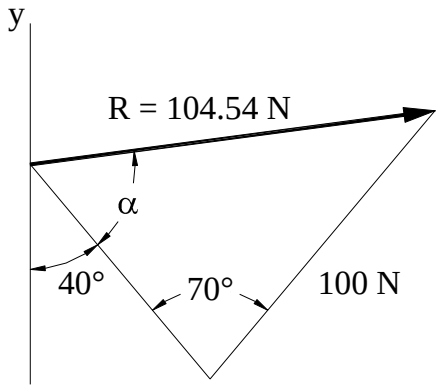
(7) Law of cosines

$$R^2 = (80\text{ N})^2 + (100\text{ N})^2 - 2(80\text{ N})(100\text{ N})\cos 70°$$

$R = 104.54$ N ←Ans.

2.1 Adding Forces by the Parallelogram Law Example 3, page 3 of 3

8 Calculate the angle that the resultant makes with the vertical.



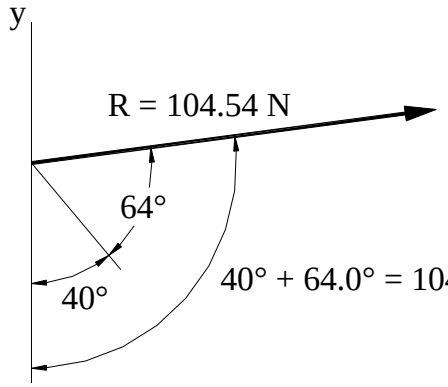
Law of sines

$$\frac{\sin \alpha}{100\ \text{N}} = \frac{\sin 70°}{\cancel{R}\ 104.54\ \text{N}}$$

Solving gives

$$\alpha = 64.0°$$

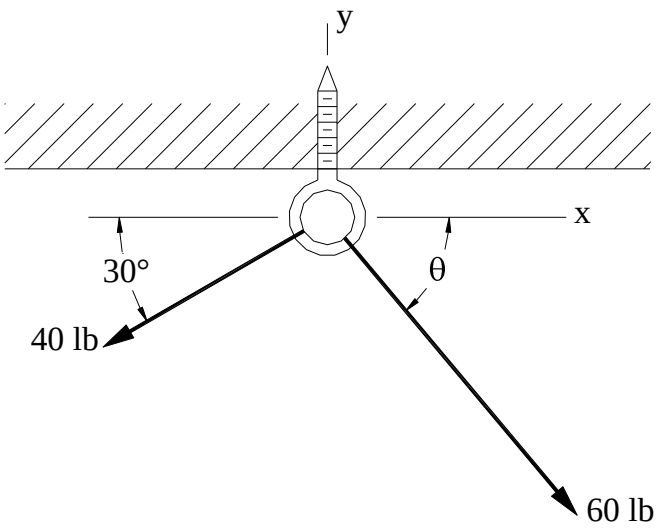
9 Angle measured from the vertical



40° + 64.0° = 104.0° ←Ans.

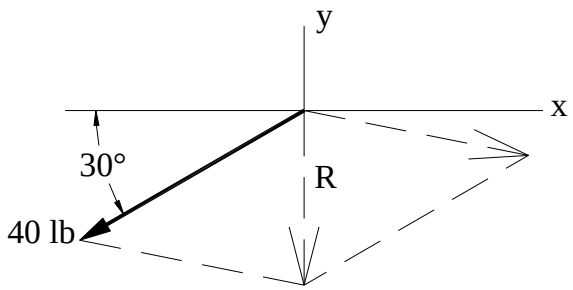
2.1 Adding Forces by the Parallelogram Law Example 4, page 1 of 4

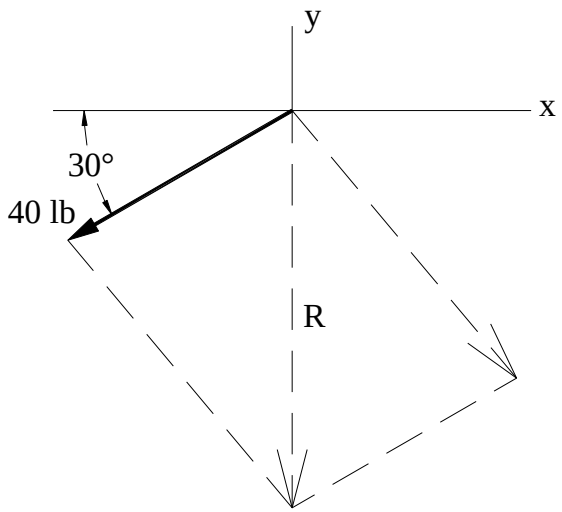
4. The resultant of the two forces acting on the screw eye is known to be vertical. Determine the angle θ and the magnitude of the resultant.

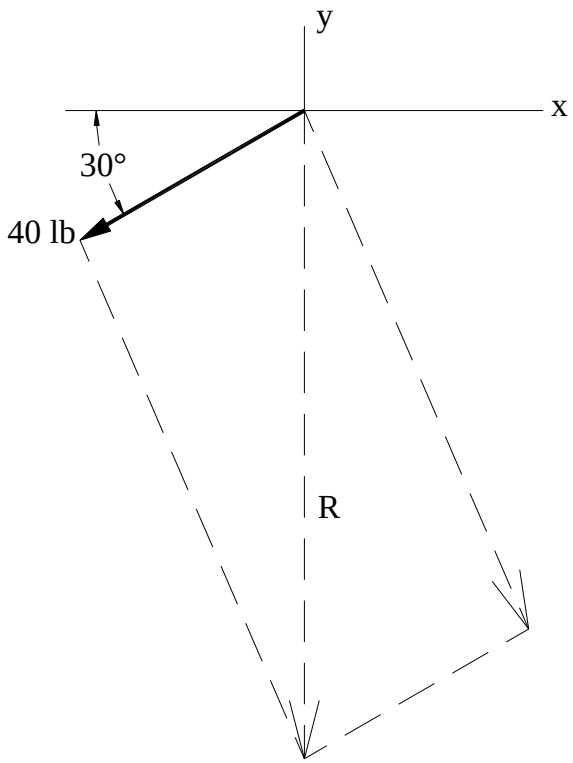


2.1 Adding Forces by the Parallelogram Law Example 4, page 2 of 4

(1) To determine what needs to be calculated, make some sketches of several possible parallelograms.







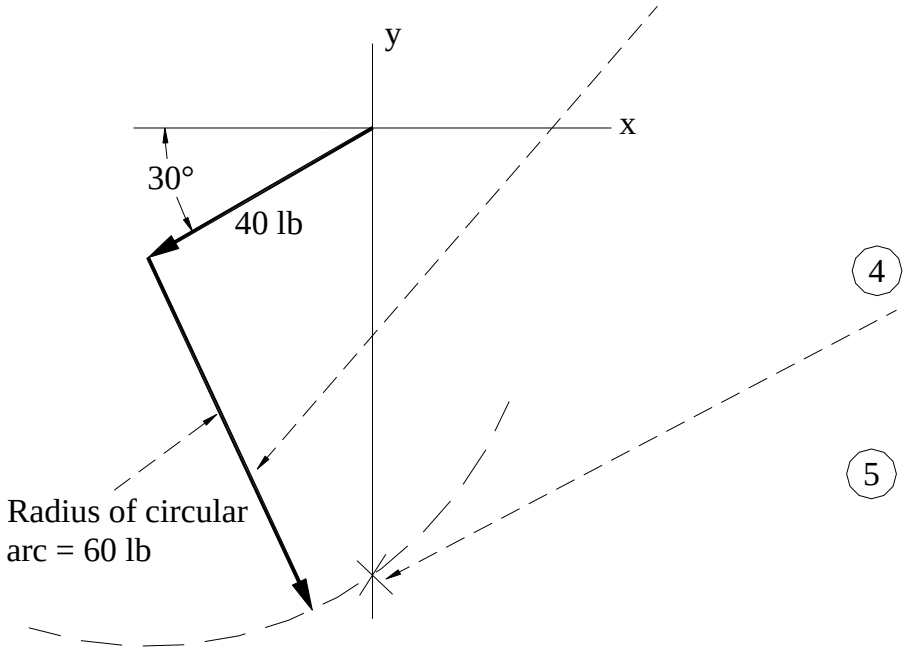
(2) Each parallelogram is based on two facts that are given:

1) One side of the parallelogram is known (40 lb at 30°), and

2) The resultant R lies on the y axis.

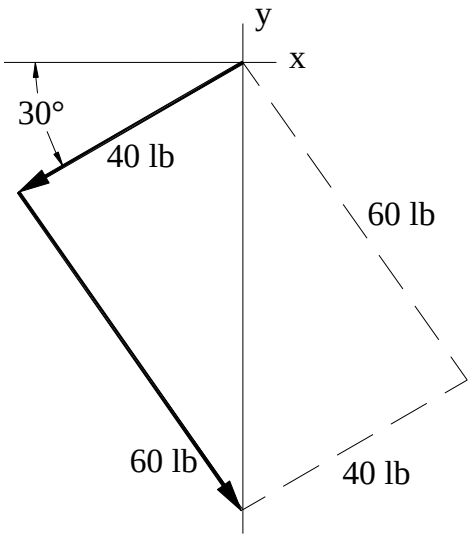
2.1 Adding Forces by the Parallelogram Law Example 4, page 3 of 4

(3) How do we determine the actual parallelogram? We have to use the additional fact that one of the forces is 60 lb.



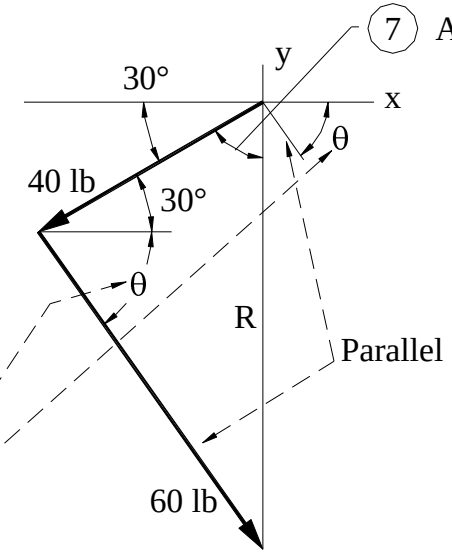
(4) The point of the intersection of the arc and the vertical axis must be the vertex of the parallelogram since it lies on the vertical axis and also lies a "distance" of 60 lb from the tip of the 40-lb vector.

(5) Now the parallelogram is completely defined.



2.1 Adding Forces by the Parallelogram Law Example 4, page 4 of 4

(6) To calculate the resultant R and and the angle θ (see below), analyze the triangle formed by the left half of the parallelogram.



(7) Angle = 90° – 30°
= 60°

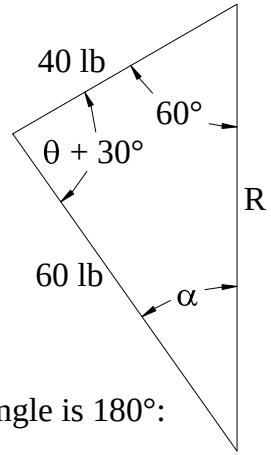
(8) Corresponding angles are equal

(9) Law of sines

$$\frac{\sin\alpha}{40\text{ lb}} = \frac{\sin 60°}{60\text{ lb}}$$

Solving gives

$$\alpha = 35.26°$$



(10) The sum of the angles of the triangle is 180°:

$$\alpha + (\theta + 30°) + 60° = 180°$$

α → 35.26°

Solving gives

$$\theta = 54.74°$$ ←Ans.

(11) Law of sines

$$\frac{\sin 60°}{60\text{ lb}} = \frac{\sin(\theta + 30°)}{R}$$

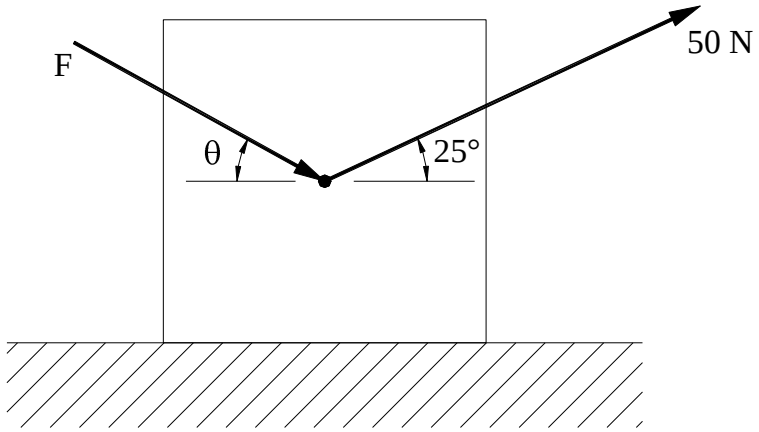
$\theta + 30°$ → 54.74°

Solving gives

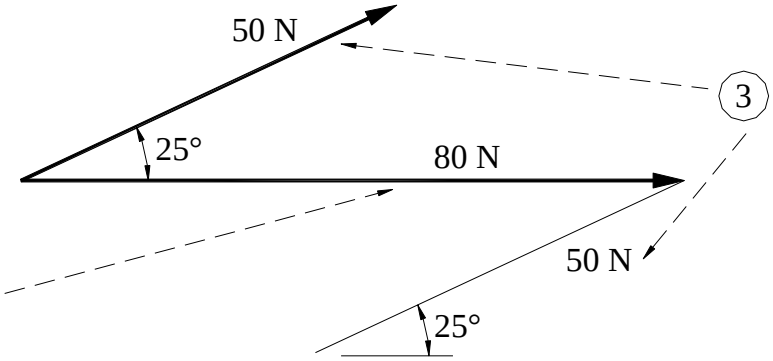
$$R = 69.0\text{ lb}$$ ←Ans.

2.1 Adding Forces by the Parallelogram Law Example 5, page 1 of 2

5. Determine the magnitude F and the angle θ, if the resultant of the two forces acting on the block is to be a horizontal 80-N force directed to the right.



(1) Draw the parts of parallelogram that are known:

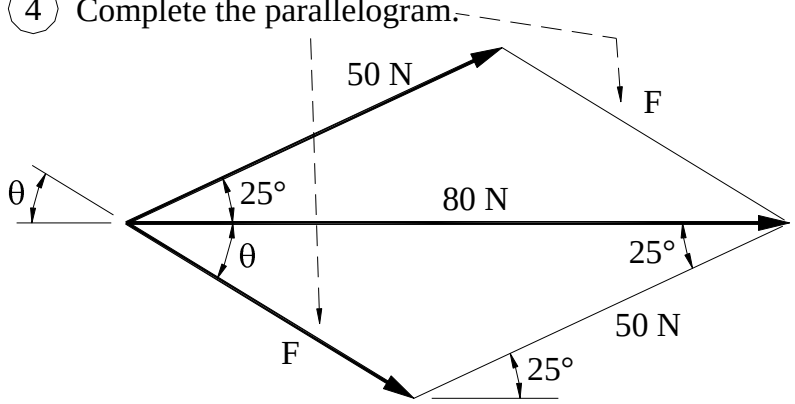


(2) The diagonal of the parallelogram (the resultant) is 80 N long and horizontal.

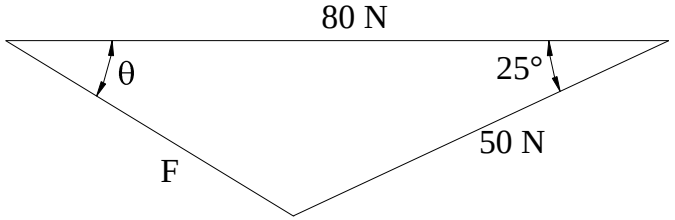
(3) Two sides are of length 50 N and make an angle of 25° with the horizontal axis.

2.1 Adding Forces by the Parallelogram Law Example 5, page 2 of 2

(4) Complete the parallelogram.



(5) Analyze the triangle forming the lower half of the parallelogram.



(6) Calculate F from the law of cosines.

$$F^2 = (50\ \text{N})^2 + (80\ \text{N})^2 - 2(50\ \text{N})(80\ \text{N}) \cos 25°$$

The result is

$$F = 40.61\ \text{N} \qquad \leftarrow \text{Ans.}$$

(7) Calculate θ from the law of sines.

$$\frac{\sin 25°}{F} = \frac{\sin \theta}{50\ \text{N}}$$

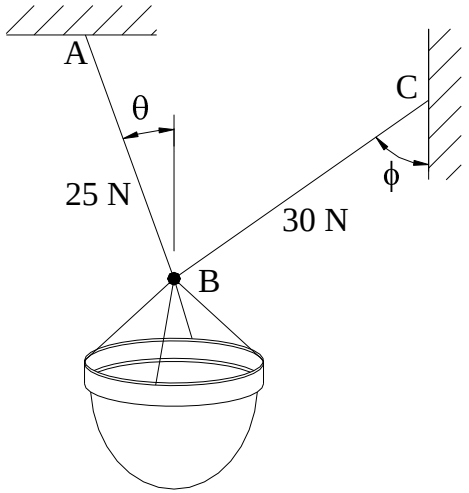
40.61 N

Solving gives

$$\theta = 31.4° \qquad \leftarrow \text{Ans.}$$

2.1 Adding Forces by the Parallelogram Law Example 6, page 1 of 2

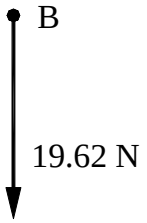
6. To support the 2-kg flower pot shown, the resultant of the two wires must point upwards and be equal in magnitude to the weight of the flower pot. Determine the angles θ and ϕ, if the forces in the wires are known to be 25 N and 30 N.



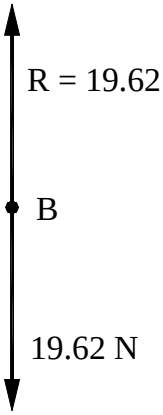
(1) Weight of flower pot

$mg = (2\text{ kg})(9.81\text{ m/s}^2)$

$= 19.62\text{ N}$

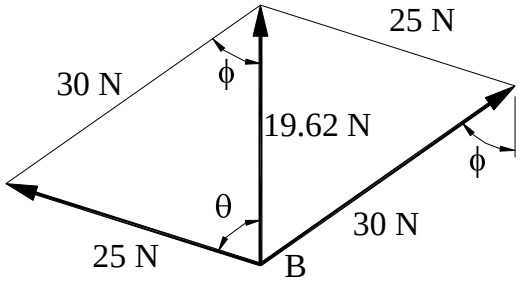


(2) Resultant, R, of forces in wires balances the weight.

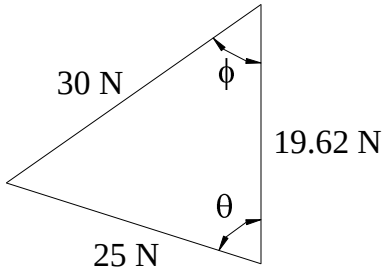


2.1 Adding Forces by the Parallelogram Law Example 6, page 2 of 2

(3) The resultant R = 19.62 N must be the diagonal of a parallelogram with sides 25 N and 30 N long.



(4) Analyze the triangle forming the left-hand half of the parallelogram.



(5) Law of cosines to calculate ϕ

$$(25\text{ N})^2 = (30\text{ N})^2 + (19.62\text{ N})^2 - 2(30\text{ N})(19.62\text{ N})\cos\phi$$

Solving gives

$$\phi = 55.90°$$ ←Ans.

(6) Law of sines to calculate θ

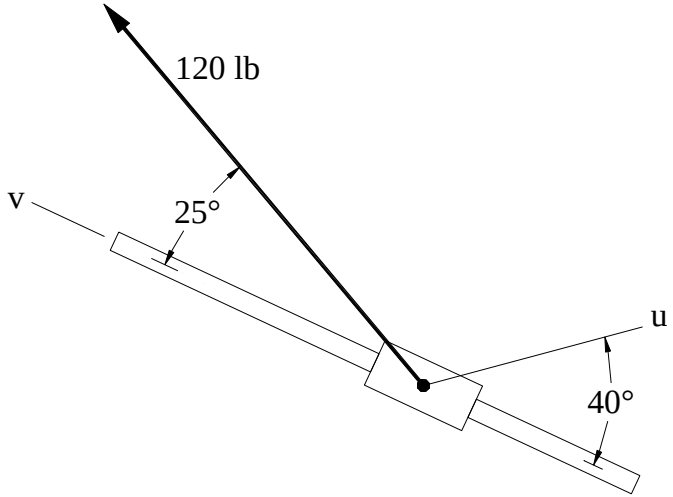
$$\frac{\sin\theta}{30\text{ N}} = \frac{\sin\phi}{25\text{ N}} \quad 55.90°$$

Solving gives

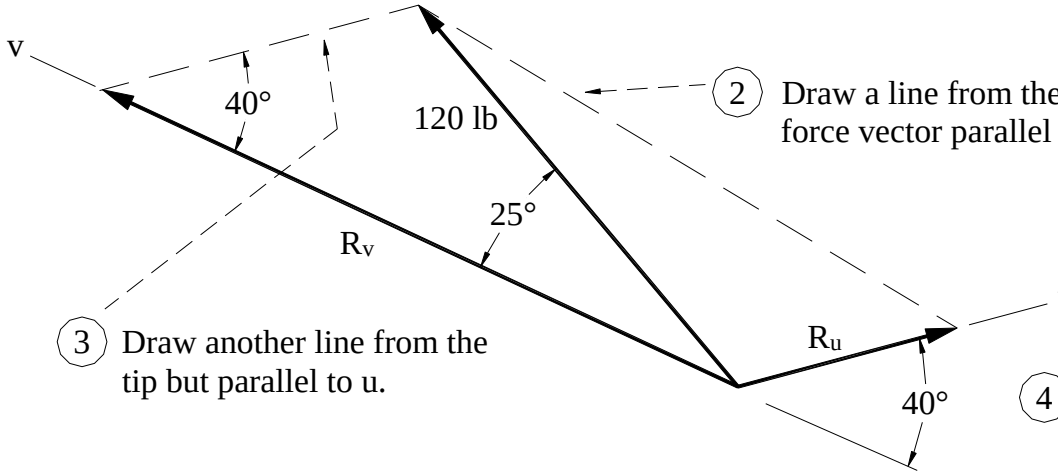
$$\theta = 83.6°$$ ←Ans.

2.1 Adding Forces by the Parallelogram Law Example 7, page 1 of 2

7. Resolve the 120-lb force into components acting in the u and v directions.



(1) Construct a parallelogram with the 120-lb force as a diagonal.



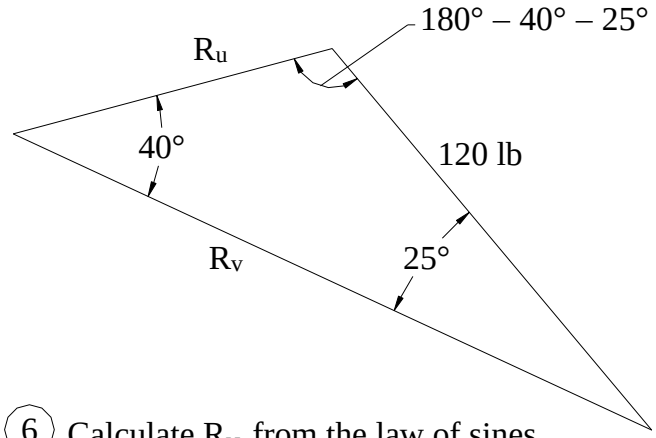
(2) Draw a line from the tip of the force vector parallel to v.

(3) Draw another line from the tip but parallel to u.

(4) Label the components R_u and R_v.

2.1 Adding Forces by the Parallelogram Law Example 7, page 2 of 2

(5) Analyze the triangle forming the left-hand half of the parallelogram.



(6) Calculate R_u from the law of sines.

$$\frac{\sin 25°}{R_u} = \frac{\sin 40°}{120 \text{ lb}}$$

Solving gives

$R_u = 78.9$ lb ←Ans.

(7) Calculate R_v from the law of sines.

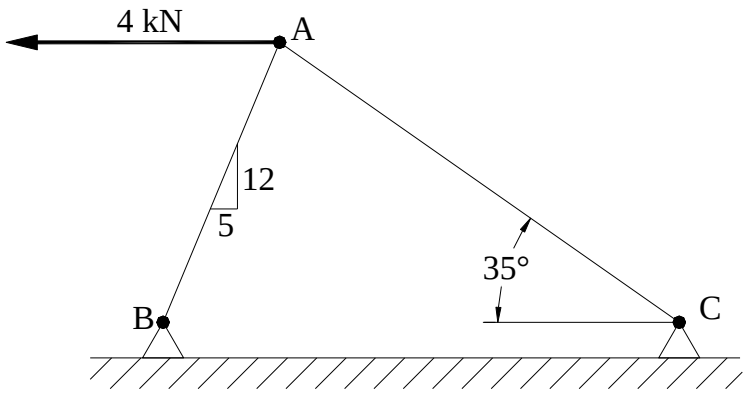
$$\frac{\sin 115°}{R_v} = \frac{\sin 40°}{120 \text{ lb}}$$

Solving gives

$R_v = 169.2$ lb ←Ans.

2.1 Adding Forces by the Parallelogram Law Example 8, page 1 of 2

8. Resolve the 4-kN horizontal force into components along truss members AB and AC.

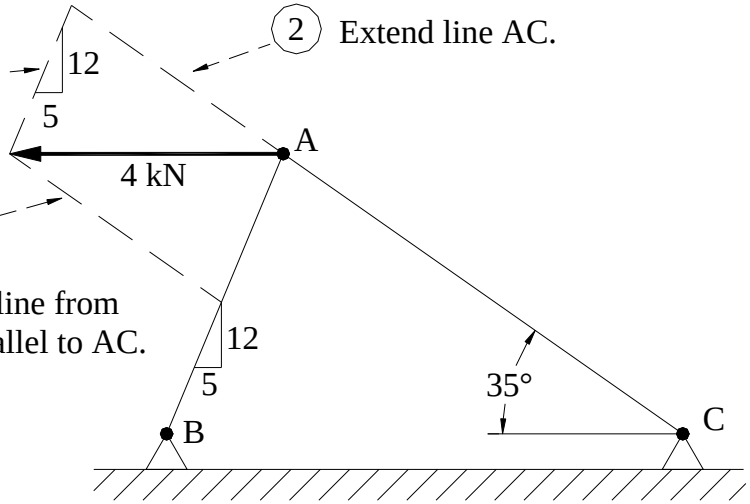


1. Construct a parallelogram with the 4-kN force as the diagonal.

2. Extend line AC.

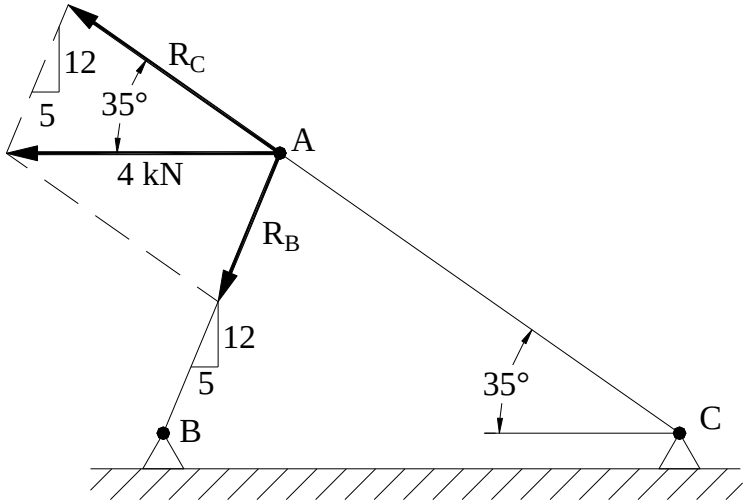
3. Draw a line from the tip of the force vector parallel to AB.

4. Draw another line from the tip but parallel to AC.

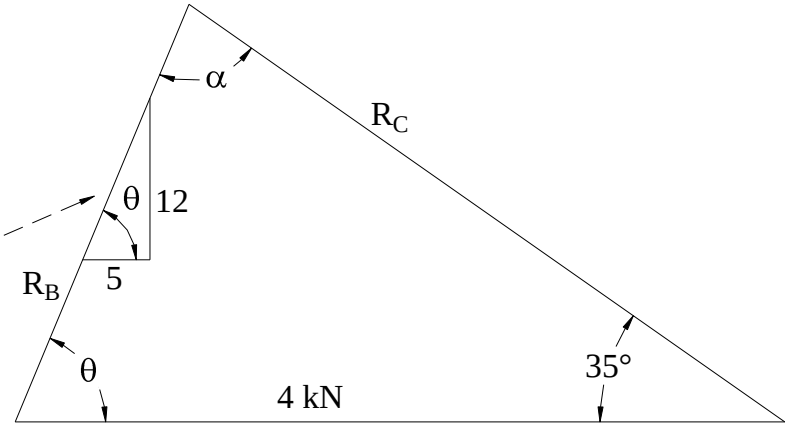


2.1 Adding Forces by the Parallelogram Law Example 8, page 2 of 2

(5) Label the components R_B and R_C.



(6) Analyze the triangle forming the upper half of the parallelogram (The drawing has been enlarged for clarity).



(7) Geometry

$$\theta = \tan^{-1}\frac{12}{5} = 67.38°$$

$$\alpha = 180° - 35° - 67.38°$$

$$= 77.62°$$

(8) Law of sines to calculate R_C

$$\frac{\sin\theta}{R_C} = \frac{\sin\alpha}{4\text{ kN}}$$

67.38° → $\sin\theta$; $\sin\alpha$ → 77.62°

Solving gives

$$R_C = 3.78\text{ kN} \quad \leftarrow\text{Ans.}$$

(9) Law of sines to calculate R_B

$$\frac{\sin 35°}{R_B} = \frac{\sin\alpha}{4\text{ kN}}$$

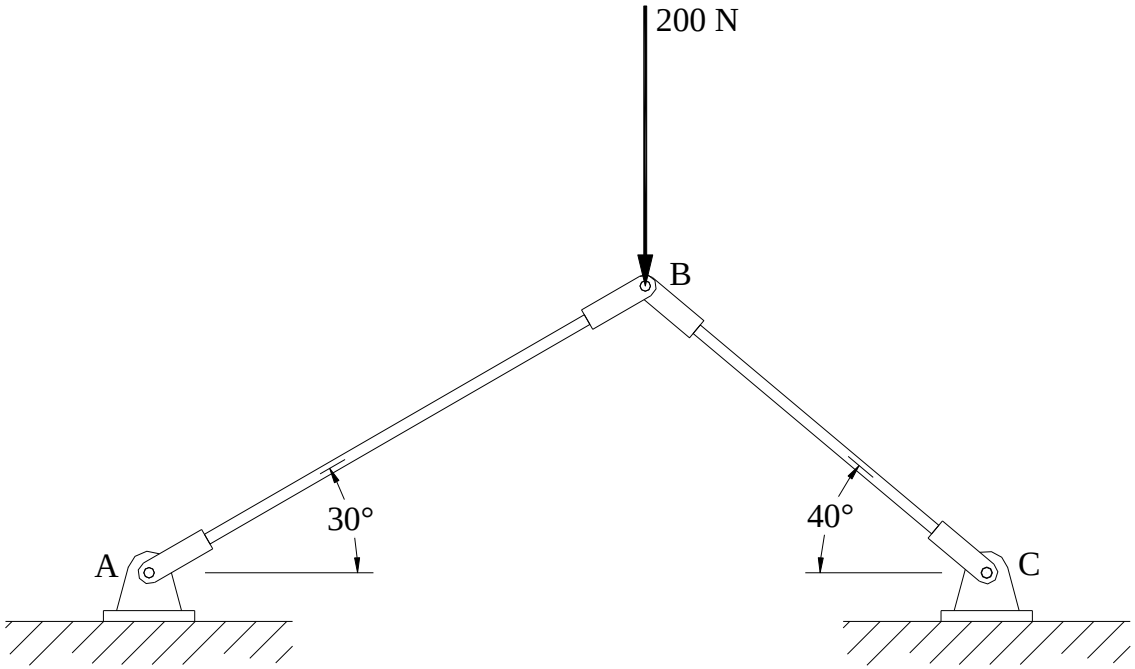
$\sin\alpha$ → 77.62°

Solving gives

$$R_B = 2.35\text{ kN} \quad \leftarrow\text{Ans.}$$

2.1 Adding Forces by the Parallelogram Law Example 9, page 1 of 3

9. Find two forces, one acting along rod AB and one along rod CB, which when added, are equivalent to the 200-N vertical force.



2.1 Adding Forces by the Parallelogram Law Example 9, page 2 of 3

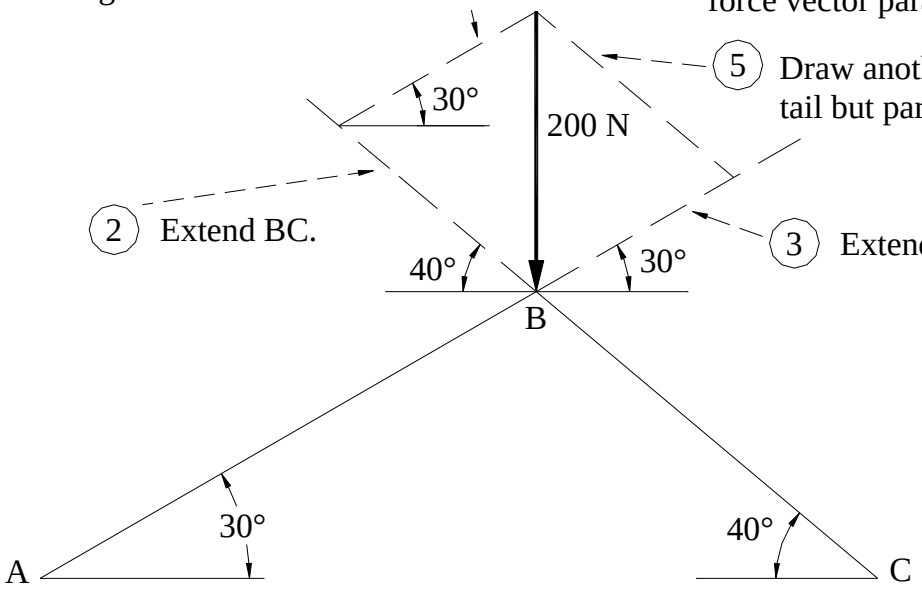
(1) Construct a parallelogram with sides parallel to AB and BC and with the 200-N force as a diagonal.

(4) Draw a line from the tail of the force vector parallel to AB.

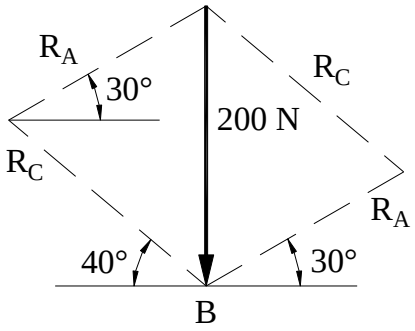
(5) Draw another line from the tail but parallel to BC.

(2) Extend BC.

(3) Extend AB.

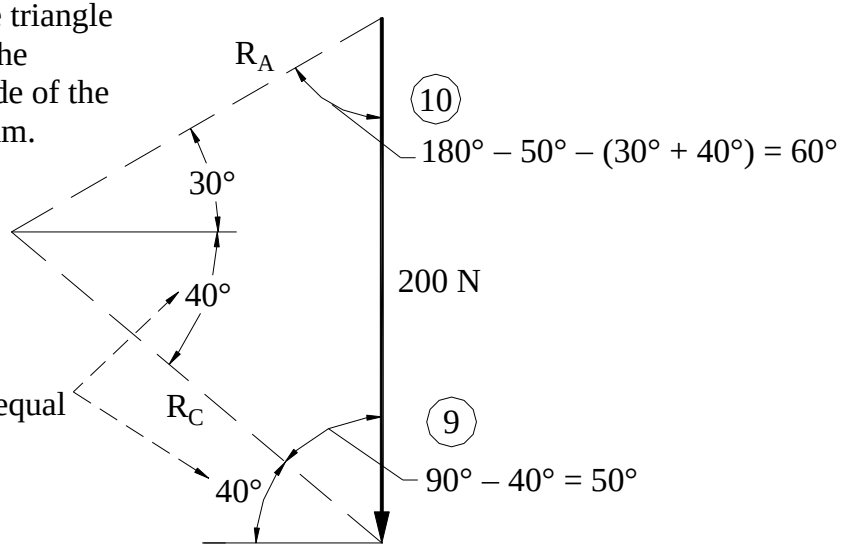


(6) Label the sides of the parallelogram R_A and R_C.



2.1 Adding Forces by the Parallelogram Law Example 9, page 3 of 3

(7) Analyze the triangle formed by the left-hand side of the parallelogram.



(8) Angles are equal

(11) Law of sines to calculate R_A

$$\frac{\sin 50°}{R_A} = \frac{\sin (30° + 40°)}{200\ \text{N}}$$

Solving gives

$R_A = 163.0$ N ←Ans.

(12) Law of sines to calculate R_C

$$\frac{\sin 60°}{R_C} = \frac{\sin (30° + 40°)}{200\ \text{N}}$$

Solving gives

$R_C = 184.3$ N ←Ans.