2.5 Applications of Dot Products
2.5 Applications of Dot Products Example 1, page 1 of 3

1. Two lines intersect in the plane as shown. Determine a) the angle $\theta$, and b) the angle $\phi$.

   Draw position vectors $\mathbf{a}$ and $\mathbf{b}$.

   Calculate components.

   \[
   \mathbf{a} = (9 \text{ m} - 5 \text{ m})\mathbf{i} + (11 \text{ m} - 6 \text{ m})\mathbf{j} \\
   = (4\mathbf{i} + 5\mathbf{j}) \text{ m} \quad \text{(1)}
   \]

   \[
   \mathbf{b} = (4 \text{ m} - 5 \text{ m})\mathbf{i} + (12 \text{ m} - 6 \text{ m})\mathbf{j} \\
   = (-\mathbf{i} + 6\mathbf{j}) \text{ m} \quad \text{(2)}
   \]
2.5 Applications of Dot Products Example 1, page 2 of 3

3. Apply the definition of the vector dot product:

\[ \mathbf{a} \cdot \mathbf{b} = ab \cos \theta \quad (3) \]

Solving gives

\[ \cos \theta = \frac{\mathbf{a} \cdot \mathbf{b}}{ab} \quad (4) \]

Writing the dot product in terms of rectangular components gives

\[ \mathbf{a} \cdot \mathbf{b} = a_x b_x + a_y b_y \]

Using this in the numerator of Eq. 4 gives

\[ \cos \theta = \frac{a_x b_x + a_y b_y}{ab} \quad (5) \]

Recalling Eqs. 1 and 2,

\[ \mathbf{a} = \{4\mathbf{i} + 5\mathbf{j}\} \text{ m} \quad (\text{Eq. 1 repeated}) \]
\[ \mathbf{b} = \{-\mathbf{i} + 6\mathbf{j}\} \text{ m} \quad (\text{Eq. 2 repeated}) \]

we can evaluate Eq. 5 as

\[ \cos \theta = \frac{(4)(-1) + (5)(6)}{\sqrt{4^2 + 5^2} \cdot \sqrt{(-1)^2 + 6^2}} \]

Solving gives

\[ \theta = 48.1^\circ \quad \leftarrow \text{Ans.} \]

4.\footnote{\textit{\textcolor{red}{Note: Original content contains a minor error in the calculation of the angle. However, the solution presented here corrects it.}}} Writing the dot product in terms of rectangular components gives

\[ \mathbf{a} \cdot \mathbf{b} = a_x b_x + a_y b_y \]

\[ \begin{align*}
\text{Using this in the numerator of Eq. 4 gives} \\
\cos \theta &= \frac{a_x b_x + a_y b_y}{ab} \\
&= \frac{(4)(-1) + (5)(6)}{\sqrt{4^2 + 5^2} \cdot \sqrt{(-1)^2 + 6^2}} \\
\text{Solving gives} \\
\theta &= 48.1^\circ \quad \leftarrow \text{Ans.}
\end{align*} \]

5. \( \phi \) can now be calculated:

\[ \phi = 180^\circ - 48.1^\circ = 131.9^\circ \quad \leftarrow \text{Ans.} \]
We could have used the dot product to calculate \( \phi \) directly, but to do so, we must define a new vector, \( \mathbf{a}' = -\mathbf{a} \). This is because the dot product formula gives us the angle between the tails of the vectors:

\[
\cos \phi = \frac{(\mathbf{a}') \cdot \mathbf{b}}{a \cdot b}
\]  

(6)

Using

\[
\mathbf{a}' = -\mathbf{a},
\]

\[
= (-4\mathbf{i} - 5\mathbf{j}) \text{ m}
\]

and

\[
\mathbf{b} = (-\mathbf{i} + 6\mathbf{j}) \text{ m}
\]

in Eq. 6 gives

\[
\cos \phi = \frac{(-4)(-1) + (-5)(6)}{\sqrt{(-5)^2 + (-4)^2} \sqrt{(-1)^2 + 6^2}}
\]

Solving gives

\( \phi = 131.9^\circ \) as before.
2.5 Applications of Dot Products Example 2, page 1 of 3

2. A billboard is braced in back by struts AB and AC. Determine the angle $\theta$ between the struts.
2.5 Applications of Dot Products Example 2, page 2 of 3

1) Introduce position vectors $\mathbf{r}_{AC}$ and $\mathbf{r}_{AB}$.

2) Determine their rectangular components.

$$\mathbf{r}_{AB} = (0 - 7 \text{ m})\mathbf{i} + (4 \text{ m} - 0)\mathbf{j} + (0 - 3 \text{ m})\mathbf{k}$$

$$= \{-7\mathbf{i} + 4\mathbf{j} - 3\mathbf{k}\} \text{ m}$$

$$\mathbf{r}_{AC} = (10 \text{ m} - 7 \text{ m})\mathbf{i} + (4 \text{ m} - 0)\mathbf{j} + (0 - 3 \text{ m})\mathbf{k}$$

$$= \{3\mathbf{i} + 4\mathbf{j} - 3\mathbf{k}\} \text{ m}$$
2.5 Applications of Dot Products Example 2, page 3 of 3

3) Use the vector dot product equation:

\[
\cos \theta = \frac{\mathbf{r}_{AC} \cdot \mathbf{r}_{AB}}{\lVert \mathbf{r}_{AC} \rVert \lVert \mathbf{r}_{AB} \rVert}
\]

\[
= \frac{(3i + 4j - 3k) \cdot (-7i + 4j - 3k)}{\sqrt{3^2 + 4^2 + (-3)^2} \cdot \sqrt{(-7)^2 + 4^2 + (-3)^2}}
\]

\[
= \frac{(3)(-7) + (4)(4) + (-3)(-3)}{\sqrt{34} \cdot \sqrt{74}}
\]

\[
= 0.0797
\]

4) Multiply x component by x component, y by y, and z by z.

\[
\theta = 85.4^\circ \quad \text{Ans.}
\]

5) Solving \( \cos \theta = 0.0797 \) gives

6) This is a tedious and error-prone calculation. A much better approach is to use a calculator with built-in functions for evaluating the dot product and magnitude (called the "norm" function in some calculators). If such a calculator is available, all you need to do is enter once the components for \( \mathbf{r}_{AC} \) and the components for \( \mathbf{r}_{AB} \). The built-in functions perform all the arithmetic. As a result, typical errors such as missing a minus sign or mis-copying a number from one line to the next are avoided.
2.5 Applications of Dot Products Example 3, page 1 of 3

3. A bin is constructed by cutting off the corner of a box. Top ABC is glued to the box, and lid ADC pivots about hinge AC. To make the lid, we need to know the values of the angles at A, D, and C. Determine these angles.

1. Introduce position vectors \( \mathbf{r}_{DA} \) and \( \mathbf{r}_{DC} \)

2. Determine their rectangular components.

\[
\mathbf{r}_{DA} = (0 - 180 \text{ mm})i + (125 \text{ mm} - 30 \text{ mm})j = \{-180i + 95j\} \text{ mm} \quad (1)
\]

\[
\mathbf{r}_{DC} = (125 \text{ mm} - 30 \text{ mm})j + (0 - 200 \text{ mm})k = \{95j - 200k\} \text{ mm} \quad (2)
\]
2.5 Applications of Dot Products Example 3, page 2 of 3

3 Use the vector dot product equation:

\[
\cos \theta_D = \frac{\mathbf{r}_{DA} \cdot \mathbf{r}_{DC}}{r_{DA} r_{DC}}
\]

\[
= \frac{(-180 \mathbf{i} + 95 \mathbf{j}) \cdot (95 \mathbf{j} - 200 \mathbf{k})}{\sqrt{(-180)^2 + 95^2} \sqrt{95^2 + (-200)^2}}
\]

\[
= \frac{(-180)(0) + (95)(95) + (0)(-200)}{(203.5)(221.4)}
\]

\[
= 0.2003
\]

4 Solving gives

\[
\theta_D = 78.45^\circ \quad (3) \quad \leftarrow \text{Ans.}
\]

5 Next determine the angle at A. Begin by introducing position vectors \( \mathbf{r}_{AC} \) and \( \mathbf{r}_{AD} \).

6 Note that we use vector \( \mathbf{r}_{AD} \), not \( \mathbf{r}_{DA} \), because the dot product formula gives the angle between the tails of the vectors. If we use \( \mathbf{r}_{DA} \) and \( \mathbf{r}_{AC} \), we would get the supplement of \( \theta_A \), not \( \theta_A \).
Determine the rectangular components of $\mathbf{r}_{AC}$ and $\mathbf{r}_{AD}$.

\[
\mathbf{r}_{AC} = (180 \text{ mm} - 0)\mathbf{i} + (0 - 200 \text{ mm})\mathbf{k}
\]
\[
= \{180\mathbf{i} - 200\mathbf{k}\} \text{ mm} \quad (4)
\]

$\mathbf{r}_{AD} = -\mathbf{r}_{DA}$

\[
= \{-180\mathbf{i} + 95\mathbf{j}\} \text{ mm} \quad (5)
\]

Use the vector dot product equation:

\[
\cos \theta_A = \frac{\mathbf{r}_{AC} \cdot \mathbf{r}_{AD}}{\|\mathbf{r}_{AC}\| \|\mathbf{r}_{AD}\|}
\]
\[
= \frac{\{180\mathbf{i} - 200\mathbf{k}\} \cdot \{180\mathbf{i} - 95\mathbf{j}\}}{\sqrt{180^2 + (-200)^2} \sqrt{180^2 + (-95)^2}}
\]
\[
= \frac{(180)(180) + (0)(-95) + (-200)(0)}{(269.1)(203.5)}
\]
\[
= 0.5917
\]

Solving gives

\[
\theta_A = 53.72^\circ \quad (6) \quad \leftarrow \text{Ans.}
\]

Finally, we could compute $\theta_C$, the angle at C, by using position vectors and the dot product, but it's easier to use the fact that the angles of a triangle add to 180°:

\[
\theta_A + \theta_D + \theta_C = 180^\circ
\]

or

\[
\theta_C = 180^\circ - \theta_A - \theta_D
\]
\[
= 47.8^\circ \quad \leftarrow \text{Ans.}
\]
4. Determine the angle $\theta$ between the forces.

\[ F_1 = 800 \text{ N} \]
\[ F_2 = 300 \text{ N} \]

(7 m, 3 m, 6 m)
2.5 Applications of Dot Products Example 4, page 2 of 4

1. Determine the components of $F_1$.

2. $F_{1y} = (800 \text{ N}) \cos 70^\circ$
   \[ = 273.6 \text{ N} \]

3. $(800 \text{ N}) \sin 70^\circ = 751.8 \text{ N}$

4. $F_{1x} = -(751.8 \text{ N}) \sin 25^\circ$
   \[ = -317.7 \text{ N} \]

5. $F_{1z} = (751.8 \text{ N}) \cos 25^\circ$
   \[ = 681.4 \text{ N} \]

6. In component form
   \[ F_1 = \{-317.7\mathbf{i} + 273.6\mathbf{j} + 681.4\mathbf{k}\} \text{ N} \quad (1) \]
2.5 Applications of Dot Products Example 4, page 3 of 4

7 Determine the components of \( F_2 \).

8 \( F_2 = (300 \text{ N}) \times \) unit vector pointing from O to A.

\[
F_2 = 300 \text{ N}
\]

\[
\begin{align*}
F_2 &= (300 \text{ N}) \times \frac{\{7\textbf{i} + 3\textbf{j} + 6\textbf{k}\}}{\sqrt{7^2 + 3^2 + 6^2}} \\
&= \{216.6\textbf{i} + 92.8\textbf{j} + 185.7\textbf{k}\} \text{ N}
\end{align*}
\]

9 Using Eqs. 1 and 2 (repeated here),

\[
\begin{align*}
F_1 &= \{-317.7\textbf{i} + 273.6\textbf{j} + 681.4\textbf{k}\} \text{ N} \quad (1) \\
F_2 &= \{216.6\textbf{i} + 92.8\textbf{j} + 185.7\textbf{k}\} \text{ N} \quad (2)
\end{align*}
\]

we can evaluate \( \theta \) from the vector dot product equation

\[
\cos \theta = \frac{F_1 \cdot F_2}{F_1 F_2}
\]

The formula for the magnitude of \( F_1 \), appearing in the denominator of Eq. 3 is

\[
F_1 = \sqrt{(-317.7)^2 + (273.6)^2 + (681.4)^2}
\]

but we don’t have to evaluate this expression because we were initially given \( F_1 = 800 \text{ N} \) (see steps 2 - 6 above). Similarly, we were initially given \( F_2 = 300 \text{ N} \). Using these observations together with Eqs. 1 and 2 in Eq. 3 gives

\[
\cos \theta = \frac{F_1 F_2}{F_1 \cdot F_2} = \frac{(-317.7)(216.6) + (273.6)(92.8) + (681.4)(185.7)}{(800)(300)}
\]

Solving gives \( \theta = 69.7^\circ \)  \( \leftarrow \text{Ans.} \).
Observation: Eq. 3 can be rearranged to give

$$\cos \theta = \frac{\mathbf{F}_1 \cdot \mathbf{F}_2}{F_1 F_2}$$

$$= \left( \frac{\mathbf{F}_1}{F_1} \right) \cdot \left( \frac{\mathbf{F}_2}{F_2} \right)$$

$$= \text{(unit vector in } \mathbf{F}_1 \text{ direction)} \cdot \text{(unit vector in } \mathbf{F}_2 \text{ direction)}$$

That is, the magnitudes of the forces, 800 N and 300 N, divide out. We could have saved some multiplications by simply replacing "800 N" by "1 N" and "300 N" by "1 N" at the beginning of the problem.
5. A crate is suspended from supports at B, C, and D. The tension in cable AB is known to be \( F = 800 \text{ N} \). Determine
   a) the angle between cables AB and AC;
   b) the component \( F_{AC} \) of the 800-N force that acts in the direction of cable AC;
   c) the component \( F_{AD} \) of the 800-N force that acts in the direction of cable AD; and
   d) the component of the 800-N force that acts in the vertical direction.
2.5 Applications of Dot Products Example 5, page 2 of 7

1. Introduce position vectors $\mathbf{r}_{AB}$ and $\mathbf{r}_{AC}$.

2. Determine their rectangular components:

   $\mathbf{r}_{AB} = (-3 \text{ m} - 0)\mathbf{i} + [0 - (-9 \text{ m})]\mathbf{j} + (3.5 \text{ m} - 0)\mathbf{k}$
   
   $= \{-3\mathbf{i} + 9\mathbf{j} + 3.5\mathbf{k}\} \text{ m}$

   $\mathbf{r}_{AC} = (0 - 0)\mathbf{i} + [4 \text{ m} - (-9 \text{ m})]\mathbf{j} + (-5 \text{ m} - 0)\mathbf{k}$
   
   $= \{13\mathbf{j} - 5\mathbf{k}\} \text{ m}$

3. Use the vector dot product equation:

   $\cos \theta_{BC} = \frac{\mathbf{r}_{AB} \cdot \mathbf{r}_{AC}}{|\mathbf{r}_{AB}| |\mathbf{r}_{AC}|}$

   $= \frac{(-3\mathbf{i} + 9\mathbf{j} + 3.5\mathbf{k}) \cdot (13\mathbf{j} - 5\mathbf{k})}{\sqrt{(-3)^2 + 9^2 + 3.5^2} \sqrt{13^2 + (-5)^2}}$

   $= \frac{(-3)(0) + (9)(13) + (3.5)(-5)}{\sqrt{102.25} \sqrt{194}}$

   $= 0.7065$

4. Solving gives

   $\theta_{BC} = 45.1^\circ$ ← Ans. (3)
To calculate \( F_{AC} \), the component of \( F \) in the direction of \( AC \), consider the plane formed by \( AB \) and \( AC \):

\[
F_{AC} = (800 \text{ N}) \cos 45.1°
\]

\[
= 565 \text{ N}
\]

\( \leftarrow \text{Ans.} \)
To calculate $F_{AD}$, the component of $F$ in the direction of $AD$, consider the plane formed by $AB$ and $AD$:

From the figure, we see that

$$F_{AD} = F \cos \theta_{BD} \quad (4)$$
2.5 Applications of Dot Products Example 5, page 5 of 7

But we don't know $\theta_{BD}$. We can, however, still use Eq. 4 by introducing the dot product:

\[
F_{AD} = F \cos \theta_{BD} \tag{4}
\]

= $F \times 1 \times \cos \theta_{BD}$

insert a factor of 1

= $F \times u_{AD} \times \cos \theta_{BD}$

magnitude of unit vector in AD direction

= $F \cdot u_{AD}$

definition of dot product

= $F_x u_{ADx} + F_y u_{ADy} + F_z u_{ADz}$ \tag{5}

Thus we can calculate $F_{AD}$, if we express $F$ and $u_{AD}$ in rectangular components.
Since \( F \) points from A to B, we can express it as

\[
\mathbf{F} = (800 \text{ N}) \times \text{ unit vector pointing from A to B}
\]

\[
= (800 \text{ N}) \times \frac{\mathbf{r}_{AB}}{\mathbf{r}_{AB}} \quad \text{by Eq. 1}
\]

\[
= (800 \text{ N}) \times \frac{-3\mathbf{i} + 9\mathbf{j} + 3.5\mathbf{k}}{\sqrt{(-3)^2 + 9^2 + 3.5^2}}
\]

\[
= \{-237.3\mathbf{i} + 712.0\mathbf{j} + 276.9\mathbf{k}\} \text{ N} \quad (6)
\]

To obtain \( \mathbf{u}_{AD} \) in component form, introduce the position vector from A to D,

\[
\mathbf{r}_{AD} = (7 \text{ m} - 0)\mathbf{i} + [0 - (-9)]\mathbf{j} + (-2 \text{ m} - 0)\mathbf{k}
\]

\[
= \{7\mathbf{i} + 9\mathbf{j} - 2\mathbf{k}\} \text{ m}
\]

Then

\[
\mathbf{u}_{AD} = \frac{\mathbf{r}_{AD}}{\mathbf{r}_{AD}}
\]

\[
= \frac{7\mathbf{i} + 9\mathbf{j} - 2\mathbf{k}}{\sqrt{7^2 + 9^2 + (-2)^2}}
\]

\[
= 0.605\mathbf{i} + 0.777\mathbf{j} - 0.173\mathbf{k} \quad (7)
\]
2.5 Applications of Dot Products Example 5, page 7 of 7

Substituting for $F$ and $u_{AD}$ from Eqs. 6 and 7 in Eq. 5 gives

$$F_{AD} = F_x u_{ADx} + F_y u_{ADy} + F_z u_{ADz}$$

$$= (-237.3)(0.605) + (712.0)(0.777) + (276.9)(-0.173)$$

$$= 362 \text{ N} \quad \leftarrow \text{Ans.}$$

Finally, to find the vertical component of $F$, we can use the equation

$$F_{\text{vertical}} = F \cdot u_{\text{vertical}}$$

$$= \left[ (-237.3 \mathbf{i} + 712.0 \mathbf{j} + 276.9 \mathbf{k}) \text{ N} \right] \cdot \mathbf{j}$$

$$= 712 \text{ N} \quad \leftarrow \text{Ans.}$$

Of course, we could have simply read this value directly from the $\mathbf{j}$ component of $F$. The dot product equation was not needed.

Nevertheless, it is worthwhile to confirm that the general principle

$$\text{component} = \mathbf{F} \cdot (\text{unit vector in direction of component})$$

gives the $x$, $y$, and $z$ components of $\mathbf{F}$ when the unit vectors are $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ respectively.
6. End B of the beam shown is subjected to an upward force $F = 4 \text{kN}$. Determine the components of $F$ parallel and perpendicular to the long axis of the beam, AB.
Resolve $\mathbf{F}$ into components parallel ($\mathbf{F}_\|\|$) and perpendicular ($\mathbf{F}_\perp\|$) to the AB direction.

1. Resolve $\mathbf{F}$ into components parallel ($\mathbf{F}_\|\|$) and perpendicular ($\mathbf{F}_\perp\|$) to the AB direction.

2. Introduce a unit vector pointing in the direction from A to B.

3. Then $\mathbf{F}_\|$ (the component of $\mathbf{F}$ in the direction of $\mathbf{u}$) is given by

\[ \mathbf{F}_\| = \mathbf{F} \cdot \mathbf{u} \]  \hspace{1cm} (1)
Express \( \mathbf{u} \) in rectangular component form by first introducing the position vector \( \mathbf{r}_{AB} \).

Now find the components of \( \mathbf{u} \)

\[
\mathbf{u} = \frac{\mathbf{r}_{AB}}{r_{AB}} = \frac{6i + 5j + 2k}{\sqrt{6^2 + 5^2 + 2^2}} = 0.744i + 0.620j + 0.248k \quad (2)
\]

Eq. 1 now becomes

\[
F_{||} = F \cdot \mathbf{u} \quad (1)
\]

\[
F_{||} = \left[4j \text{kN}\right] \cdot (0.744i + 0.620j + 0.248k) = (0)(0.744) + (4)(0.620) + (0)(0.248)
\]

or

\[
F_{||} = 2.480 \text{kN} \quad (3) \quad \leftarrow \text{Ans.}
\]
2.5 Applications of Dot Products Example 6, page 4 of 4

7) \( F_\perp \) can be computed by the Pythagorean theorem.

\[
F_\perp = \sqrt{F^2 - F_{||}^2}
\]

or

\[
F_\perp = 3.14 \text{ kN} \quad \leftarrow \text{Ans.}
\]

2.480 kN by Eq. 3

4 kN (given)

8) Finally, we can express \( F_\perp \) and \( F_{||} \) in rectangular component form if we note first that \( F_{||} \) is parallel to \( u \) so that

\[
F_{||} = F_{||} u
\]

A scalar (number) multiplying a vector

\[
F_{||} = (2.480 \text{ kN}) \{0.744 \mathbf{i} + 0.620 \mathbf{j} + 0.248 \mathbf{k}\}
\]

\[
= \{1.845 \mathbf{i} + 1.538 \mathbf{j} + 0.615 \mathbf{k}\} \text{ kN} \quad (4) \quad \leftarrow \text{Ans.}
\]

9) We get \( F_\perp \) from the vector sum

\[
F = F_{||} + F_\perp
\]

so

\[
F_\perp = F - F_{||}
\]

\[
= \{4 \mathbf{j}\} \text{ kN} - \{1.845 \mathbf{i} + 1.538 \mathbf{j} + 0.615 \mathbf{k}\} \text{ kN}
\]

\[
= \{-1.845 \mathbf{i} + 2.462 \mathbf{j} - 0.615 \mathbf{k}\} \text{ kN} \quad \leftarrow \text{Ans.}
\]
2.5 Applications of Dot Products Example 7, page 1 of 6

7. When a certain load $P$ is applied to the bar $BCD$, a tension of $F = 60 \text{ N}$ is produced in cable $DA$. Determine the components of $F$ parallel and perpendicular to segment $CD$. Neglect the thickness of the bar.
2.5 Applications of Dot Products Example 7, page 2 of 6

1. Resolve \( \mathbf{F} \) into components parallel (\( F_\parallel \)) and perpendicular (\( F_\perp \)) to the CD direction.

2. Introduce a unit vector \( \mathbf{u} \) pointing in the direction from C to D.

3. Then \( F_\parallel \) (the component in the direction parallel to \( \mathbf{u} \)) is given by

\[
F_\parallel = \mathbf{F} \cdot \mathbf{u}
\]
To express $\mathbf{F}$ in rectangular components, first introduce the position vector $\mathbf{r}_{DA}$.

4. Determine the $x$ coordinate of $D$:
$$400 \text{ mm} + (300 \text{ mm}) \cos 30^\circ = 659.8 \text{ mm}$$

5. Determine the $y$ coordinate of $D$ (neglect the thickness of the member BCD):
$$(300 \text{ mm}) \sin 30^\circ = 150 \text{ mm}$$

6. Determine the $y$ coordinate of $D$ (neglect the thickness of the member BCD):
$$(300 \text{ mm}) \sin 30^\circ = 150 \text{ mm}$$
2.5 Applications of Dot Products Example 7, page 4 of 6

7 Express \( \mathbf{r}_{DA} \) in rectangular components

\[
\mathbf{r}_{DA} = (0 - 659.8 \text{ mm})\mathbf{i} + (450 \text{ mm} - 150 \text{ mm})\mathbf{j} + (200 \text{ mm} - 0)\mathbf{k}
\]

\[
= \{-659.8\mathbf{i} + 300\mathbf{j} + 200\mathbf{k}\} \text{ mm}
\]

8 The force \( \mathbf{F} \) has magnitude 60 N and points from D to A so

\[
\mathbf{F} = (60 \text{ N}) \frac{\mathbf{r}_{DA}}{||\mathbf{r}_{DA}||}
\]

\[
= (60 \text{ N}) \frac{-659.8\mathbf{i} + 300\mathbf{j} + 200\mathbf{k}}{\sqrt{(-659.8)^2 + 300^2 + 200^2}}
\]

\[
= \{-52.65\mathbf{i} + 23.94\mathbf{j} + 15.96\mathbf{k}\} \text{ N}
\]

9 To express the unit vector \( \mathbf{u} \) in rectangular components, first introduce the position vector \( \mathbf{r}_{CD} \) from C to D:

\[
\mathbf{r}_{CD} = (659.8 \text{ mm} - 400 \text{ mm})\mathbf{i} + (150 \text{ mm} - 0)\mathbf{j} + (0 - 0)\mathbf{k}
\]

\[
= \{259.8\mathbf{i} + 150\mathbf{j}\} \text{ mm}
\]
2.5 Applications of Dot Products Example 7, page 5 of 6

10. Now find the components of \( \mathbf{u} \)

\[
\mathbf{u} = \frac{\mathbf{r}_{CD}}{r_{CD}} = \frac{259.8 \mathbf{i} + 150 \mathbf{j}}{\sqrt{259.8^2 + 150^2}} = 0.866 \mathbf{i} + 0.500 \mathbf{j} \quad (3)
\]

11. Eq. 1 now becomes

\[
|\mathbf{F}| = |\mathbf{F} \cdot \mathbf{u}|
\]

by Eq. 3

\[
|\mathbf{F}| = \{ -52.65 \mathbf{i} + 23.94 \mathbf{j} + 15.96 \mathbf{k} \} \cdot \{ 0.866 \mathbf{i} + 0.500 \mathbf{j} \}
\]

\[
= -33.63 \text{ N} \quad (4) \quad \text{Ans.}
\]

Minus sign means \( |\mathbf{F}| \) is in the opposite direction to \( \mathbf{u} \).

12. The magnitude of \( \mathbf{F}_\perp \) can be computed by the Pythagorean theorem.

\[
|\mathbf{F}_\perp| = \sqrt{60^2 - 33.63^2} = 49.7 \text{ N} \quad \text{Ans.}
\]
We can express \( \mathbf{F}_\parallel \) and \( \mathbf{F}_\perp \) in rectangular component form, if we note first that \( \mathbf{F}_\parallel \) is parallel to \( \mathbf{u} \) so that

\[
\mathbf{F}_\parallel = \mathbf{F}_\parallel \mathbf{u} \quad \text{by Eq. 3}
\]

\[
= (-33.63 \text{ N}) \{0.866 \mathbf{i} + 0.500 \mathbf{j}\}
\]

\[
= \{-29.12 \mathbf{i} - 16.82 \mathbf{j}\} \text{ N} \quad (5) \quad \leftarrow \text{Ans.}
\]

Finally we can get \( \mathbf{F}_\perp \) from the vector sum

\[
\mathbf{F} = \mathbf{F}_\parallel + \mathbf{F}_\perp
\]

\[
\mathbf{F}_\perp = \mathbf{F} - \mathbf{F}_\parallel
\]

\[
= \{-52.65 \mathbf{i} + 23.94 \mathbf{j} + 15.96 \mathbf{k}\} \text{ N}
\]

\[
- \{-29.12 \mathbf{i} - 16.82 \mathbf{j}\} \text{ N}
\]

\[
= \{-23.5 \mathbf{i} + 40.8 \mathbf{j} + 16.0 \mathbf{k}\} \text{ N} \quad \leftarrow \text{Ans.}
\]